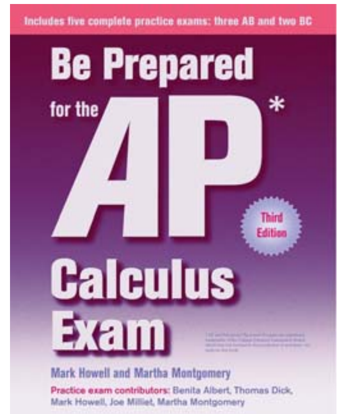
Be Prepared

for the

AP

Calculus Exam



Third Edition

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Chapter 10. Annotated Solutions to Past Free-Response Questions

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The free-response questions for this exam are posted on AP Central, apcentral.collegeboard.org:

- AP Calculus AB
- AP Calculus BC

2026 AP Calculus AB

Free-Response Solutions and Notes

Question AB-1

A. $M'(7.5) \approx \dfrac{M(10)-M(5)}{10-5} = \dfrac{16-7}{10-5} = \dfrac{9}{5}$ birds per day per day.[1]

B. i. $\int_0^{30} M(t)\,dt \approx 10 \cdot M(5) + 10 \cdot M(15) + 10 \cdot M(25) = 10 \cdot 7 + 10 \cdot 6 + 10 \cdot 2 = 150$. [2]

ii. $\int_0^{30} M(t)\,dt$ is the number of male birds that arrived at the nesting area over the 30 day period.

C. $\int_{15}^{45} 18 + 16\sin\left(\dfrac{\pi}{20}(t+15)\right)dt =$ 641.859. From $t = 15$ to $t = 45$ days, 642 female birds arrived at the nesting area.

D. $D(15) = M(15) - F(15) = 6 - 2 = 4 > 0$ and
$D(20) = M(20) - F(20) \approx 5 - 6.686 = -1.686 < 0$. Since $D(t)$ is differentiable on the interval $(15, 20)$, it must also be continuous [3] and the Intermediate Value Theorem guarantees a time *t* in the interval (15, 20) when $D(t) = 0$.

Notes:

1. You must show the quotient of differences for credit, but the expression $\dfrac{16-7}{10-5}$ need not be simplified.
2. You must show the sum of products for credit, but the expression $10 \cdot 7 + 10 \cdot 6 + 10 \cdot 2$ need not be simplified.
3. You must state the continuity is a consequence of differentiability in the justification. It is not necessary to cite the Intermediate Value Theorem by name, but it is essential that you state the hypothesis and conclusion as applied to this situation.

Question AB-2

A. $\int_0^1 g(x)\,dx =$ 🖩 1.513. [1]

B. Every cross section of the solid has base $g(x)$ and height $\frac{1}{3}g(x)$, so the area of a cross section is $A(x) = \frac{1}{3}\left(g(x)\right)^2$ and the volume is $\int_0^1 A(x)\,dx = \frac{1}{3}\int_0^1 \left(g(x)\right)^2\,dx$.

C. $f(x) = g(x) \;\Rightarrow\; x = 1$ and $x = 3.25582$. Let a = 3.25582. The area is $\int_0^1 f(x) - g(x)\,dx + \int_1^a g(x) - f(x)\,dx =$ 🖩 0.632.

D. Every cross section of the solid is a circle with radius $h(y)$, so the area of a cross section is $A(y) = \pi\left(h(y)\right)^2$ and the volume is $\int_1^{3.5} A(y)\,dy = \pi\int_1^{3.5} \left(h(y)\right)^2\,dy$.

Notes:

1. Since the function $g(x)$ is needed later in the problem, store the function in the calculator, and use the stored function in all the calculations. This is more efficient than retyping the function definition in all four parts, and helps avoid transcription errors.

Question AB-3

A. The slope field shows positive slopes for $H \ge 30$, but when $H \ge 30$, $\frac{dH}{dt} < 0$.

B. The slope is $-\frac{1}{15}(75-20) = -\frac{55}{15} = -\frac{11}{3}$. 1

C. Since $20 < H(t) < 75$, $\frac{d^2H}{dt^2} > 0$ for $0 < t < 5$. Therefore, the tangent line gives an underestimate for $H(5)$.

D. $\frac{dH}{H-20} = -\frac{1}{15}dt \Rightarrow \ln(H-20) = -\frac{1}{15}t + C$. Substituting the initial condition, $\ln(55) = C \Rightarrow \ln(H-20) = -\frac{1}{15}t + \ln(55) \Rightarrow H = 20 + 55e^{-t/15}$.

Notes:

1. You could leave the answer as $-\frac{1}{15}(75-20)$.

Question AB-4

A. $g'(x) = f'(x) - \frac{1}{x}; g'(2) = f'(2) - \frac{1}{2} = 1.5 - \frac{1}{2} = 1$.

B. The graph of *f* has only one point of inflection, at $x = 1$, because $f'(x)$ has an extremum at this point. [1]

C. The graph of *f* is increasing and concave down for $1 < x < 3$ because $f'(x) > 0$ and $f'(x)$ is decreasing on that interval.

D. Since $f'(x) < 0$ for $-4 < x < -2$ and $f'(x) \geq 0$ for $-2 < x < 4$, *f* has its absolute minimum at $x = -2$. *f* decreases by $\int_{-4}^{-2} |f'(x)|\,dx$ on the interval [–4, –2] and increases by $\int_{-2}^{4} f'(x)\,dx$ on the interval [–2, 4]. Since $\int_{-4}^{-2} |f'(x)|\,dx < \int_{-2}^{4} f'(x)\,dx$, *f* reaches its absolute maximum at $x = 4$.

Notes:

1. In this setting, points of inflection can be justified in other ways. For example, you could assert that f' changes from increasing to decreasing at $x = 1$.

Question AB-5

A. $a(1) = v'(1) = 4 \cdot 1^3 - 24 \cdot 1^2 + 32 \cdot 1 = 12$.

B. $v(1) = 1 - 8 + 16 > 0$ and $a(1) > 0$, so the car is speeding up.

C. As shown in the graph, [1] the velocity is non-negative for $0 \le t \le 4$. The distance traveled is $\int_0^4 \left(t^4 - 8t^3 + 16t^2\right) dt = \left(\frac{t^5}{5} - 2t^4 + \frac{16t^3}{3}\right)\Bigg|_0^4 = \frac{4^5}{5} - 2 \cdot 4^4 + \frac{16 \cdot 4^3}{3}$. [2]

D. $\frac{1}{12-6}\int_6^{12} 10\cos\left(\frac{\pi}{3}t\right) - 10dt = \frac{1}{6}\left(\frac{30}{\pi}\sin\left(\frac{\pi}{3}t\right) - 10t\right)\Bigg|_6^{12} =$

$= \frac{1}{6}\left(\left(0-120\right) - \left(0-60\right)\right) = -10$

Notes:

1. $\left(t^4 - 8t^3 + 16t^2\right) = t^2\left(t^2 - 8t + 16\right) = t^2\left(t-4\right)^2$ verifies that $v(t) \ge 0$ for $0 \le t \le 4$.
2. Leave the answer unsimplified. For the record, though, this is $\frac{512}{15}$.

Question AB-6

A. Since f is twice differentiable, it must also be continuous, and we can evaluate the limit by direct substitution: $\lim\limits_{x\to 2}\dfrac{f(x)}{x}=\dfrac{f(2)}{2}=\dfrac{3}{2}$.

B. $g'(2)=f'\big(f(2)\big)f'(2)=9\cdot 4=36$.

C. $h(2)=h(0)+\int_0^2 h'(x)\,dx=10+\int_0^2 f'(3x)\,dx=10+\dfrac{1}{3}f(3x)\Big|_0^2=10+\dfrac{5+1}{3}=12$. [1]

D. i. $k'(x)=x^2 f(x)$

ii. $k''(x)=x^2 f'(x)+2xf(x) \;\Rightarrow\; k''(3)=9f'(3)+6f(3)=81+48=129$. [1]

Notes:

1. As always, it is not necessary to simplify expressions that can be evaluated with a scientific calculator.

2026 AP Calculus BC
Free-Response Solutions and Notes

Question BC-1

See AB Question 1.

Question BC-2

A. The area is $\frac{1}{2}\int_0^{\pi}\left(r(\theta)\right)^2\,d\theta = 18.064$. [1]

B. $-\frac{3}{7} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{3\sqrt{2}}{2}}{dx/d\theta} \;\Rightarrow\; \frac{dx}{d\theta} = \frac{\frac{3\sqrt{2}}{2}}{\frac{-3}{7}} = -\frac{7\sqrt{2}}{2}$.

C. $r'(\theta) = 0 \;\Rightarrow\; \theta = 0.554.$ [2] Because $r'(\theta)$ changes sign from positive to negative at this point, r has a local maximum here.

D. Since $r(\theta) > 0$, the average distance from the origin is the average value of r. This is $\frac{1}{\pi - \frac{\pi}{2}}\int_{\pi/2}^{\pi} r(\theta)\,d\theta = 1.727$.

Notes:

1. It is a good idea to enter the function for the curve and check that your graph agrees with the graph shown in the problem. Use the stored function in subsequent calculations; this helps avoid transcription errors.
2. It takes some skill with the calculator to use the names of the functions in computations. But practicing this is well worth the effort. Here you could enter $r'(\theta)$ as a function of X, $F(X) = 4\cos(2X) - 2\sin(2X)$, then find a zero of its graph in Function mode.

Question BC-3

See AB Question 3.

Question BC-4

See AB Question 4.

Question BC-5

A. $\int_1^2 (x-1)^{1/3}\,dx = \frac{3}{4}(x-1)^{4/3}\Big|_1^2 = \frac{3}{4}$.

B. $\pi\int_1^2 (x-1)^{2/3}\,dx$.

C. $f'(x) = \frac{1}{3}(x-1)^{-2/3}$. The perimeter is $1+1+\int_1^2 \sqrt{1+\left(\frac{1}{3}(x-1)^{-2/3}\right)^2}\,dx$. [1]

D. $\int_2^\infty g(x)\,dx = \lim_{b\to\infty}\int_2^b e^{-2x+4}dx = \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2x+4}\right)\Big|_2^b = \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2b+4} - \left(-\frac{1}{2}e^{-4+4}\right)\right)$

$= 0 - \left(-\frac{1}{2}e^0\right) = \frac{1}{2}$. [2]

Notes:

1. If you have calculated and identified $f'(x)$ in your work, you could use $f'(x)$ in the arc-length integral.
2. It is essential to use correct limit notation when evaluating improper integrals.

Question BC-6

A. With $x = 3$, the common ratio is $-\frac{3}{5}$. Since the first term is 2, the sum is $\frac{2}{1-\left(-\frac{3}{5}\right)} = \frac{5}{4}$.

B. $-\frac{2}{5}+\frac{4}{25}x-\frac{6}{125}x^2+\frac{8}{625}x^3$.

C. When $x = \frac{5}{2}$, the series for f is an alternating series whose terms decrease in magnitude to 0. Therefore, the alternating series error bound can be used. So

$$\left|f\left(\frac{5}{2}\right)-\left(-\frac{3}{10}\right)\right| \le \frac{8\cdot\left(\frac{5}{2}\right)^3}{625} = \frac{1}{5}.$$ ▯1

D. i. $1+x+\frac{x^2}{2}$.

ii. The first three terms of the Maclaurin series for $25g(x)-2e^x$ are

$$50-10x+2x^2-\left(2+2x+x^2\right)=48-12x+x^2.$$

▯ **Notes:**

1. For full credit, you must communicate that the error is bounded by the first omitted term with an inequality.