Mathematics for the Digital Age and Programming in Python

>>> Second Edition:
with Python 3

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```
# This program demonstrates some elements
# of Python's syntax
# Author: H. Dumpty

def someFun(n):
    '''This function takes a positive integer n
       performs some mysterious calculations, and
       returns a positive integer'''
    k = 0
    lst = []  # empty list

    while n > 0:
        if n % 2 != 0:
            lst.append(k)  # or: lst += [k]
        k += 1    # same as: k = k + 1
        n //= 2   # integer division with truncation

    lst2 = [2 ** k for k in lst]  # list comprehension
    return sum(lst2)

n = -1

while n <= 0:
    s = input('Enter a positive integer: ')
    try:
        n = int(s)
    except ValueError:
        print('Invalid input')

r = someFun(n)
print('n =', n, end=' ')
print('r =', r)

# Three other ways to display the same output:

print('n =', n, 'r =', r)
print('n = {0:d} r = {1:d}'.format(n, r))
print('n = ' + str(n) + ' r = ' + str(r))
```

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