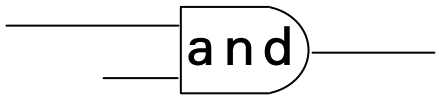
Mathematics
for the Digital Age



Programming
in Python

>>> Second Edition:
with Python 3

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Library of Congress Control Number: 2009913596

ISBN 978-0-9824775-8-8

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1 2 3 4 5 6 7 8 9 10 15 14 13 12 11 10

Printed in the United States of America

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How to Use This Book

The *Math and Python* companion web site —

`http://www.skylit.com/python`

— is an integral part of this book. It contains downloadable student files for exercises, assembled together in what we call *Student Disk*. Also on the book's web site are some of the appendices, links, errata, supplemental papers and syllabi, and technical support information for teachers.

`PY` refers to the *Math and Python Student Disk*. For example, "The file of words, `words.txt`, is provided in `PY\Ch09`" means the `words.txt` file is located in the `Ch09` subfolder in your Student Disk folder.

This icon draws your attention to a hands-on exploration of an example.

"Parentheses" like these, in the margin, mark supplementary material intended for a more inquisitive reader. This material either gives a glimpse of things to come in subsequent chapters or adds technical details.

1.▪, **2.**◆ In exercises, a square indicates an "intermediate" question that may require more thought or work than an ordinary question or exercise. A diamond indicates an "advanced" question that could be treacherous or lead to unexplored territory or take a lot of work.

✓ A checkmark at the end of a question in exercises means that the answer or a solution is included on your student disk. We have included answers and solutions to about one-third of the exercises. They can be found in `PY\SolutionsToExercises` (click on `index.html`).

Teacher Disk, which contains complete solutions to all the exercises and labs, is available for downloading free of charge to teachers who use this book as a textbook in their school. Go to `skylit.com/python` and click on the "Teachers' Room" link for details.

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Preface

"So, is this a math book or a computer programming book?" This is probably the first question on the impatient reader's mind. But why should it be? It is a librarian's dilemma: Does it go on the math shelf or on the computer shelf? There is a simple solution: put a copy on each.

The purpose of this book is to teach a particular way of thinking — precision thinking — and how to solve problems that require this way of thinking. Both mathematics and computer programming nourish the ability to think with precision and to solve problems that call for exact solutions.

Mathematics teaches us to appreciate the beauty of a rigorous argument. In the long run, this is more valuable than a lesson on solving today's practical problems. Still, mathematics does not exist in a vacuum — its abstractions are rooted in practical knowledge accumulated over centuries. The teaching of mathematics draws on examples and analogies from the world around us. At least, it should. However, the world around us is changing more and more rapidly. In the past 50 or 60 years, our world has changed dramatically: it has gone digital. This change is so profound that it is sometimes hard to fully comprehend. Is that why the change remains largely ignored in our K-12 math curricula? We need to start filling the gap.

If we could build a time machine and bring Euclid over for a visit, he would find it comforting amid the chaos of modern technologies that geometry familiar to him is still taught in schools. Old rivals Newton and Leibniz would both find great satisfaction in the fact that tens of thousands of 11th and 12th graders are learning how to take derivatives and use integrals. But George Boole, a visitor from the more recent past, would have to search through dozens of school textbooks before he could find his algebra of propositions mentioned even in passing, despite the fact that his name is immortalized in every modern computer programming language. As for John von Neumann, a brilliant mathematician and one of the fathers of computer technology... well, with his usual optimism he would predict that within 20 years or so, every elementary school student will learn about the AND, OR, and NOT gates. And why not?

In this book we have collected some of the easier mathematical topics that are relevant to the digital world. Many of these topics are often bundled together in

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freshman college courses under the name *discrete mathematics*. Discrete mathematics has become a euphemism for all elementary mathematics that is relevant today but neglected in standard middle and high school algebra, precalculus, and calculus courses. In the 1970s, Donald Knuth and his colleagues at Stanford coined the phrase "concrete mathematics" — a blend of CONtinuous and disCRETE mathematics (and also solid and not too abstract) — to describe the course Knuth taught at Stanford. Later, *Concrete Mathematics* became the title of their delightful book.[1] As they explain in the preface, Knuth "had found that there were mathematical tools missing from his repertoire; the mathematics he needed for a thorough, well grounded understanding of computer programs was quite different from what he'd learned as a mathematics major in college."

We believe that starting in college is too late. Many concepts are completely accessible to middle and high school students. And there is also another side to the relationship: just as mathematics helps achieve a deeper understanding of computer programs, some hands-on experience with computer programming helps make mathematics more tangible, familiar, and easier to grasp.

So, if you are interested mainly in computers, we hope this book will make you a better computer programmer. If you are more interested in math, you will have ample opportunities to solve interesting problems and model some of them in computer programs. You will become familiar with fun areas of mathematics that are usually kept from middle and high school students for no obvious reason; you will learn to solve real problems (that is, problems that you don't already know how to "solve" ahead of time); you will learn the power of mathematical reasoning and proof. As a bonus, you will acquire the practical skill of programming in Python, a popular commercial programming language.

We chose Python for several reasons. First, Python gives you a chance to experiment with the language in an interactive setting with immediate feedback. Second, Python's syntax is not too complicated. Third, Python has simple but powerful features for working with lists and "dictionaries" (maps). Finally, Python is easy to install and get started with, and it's free. Of course, there are other programming languages that have similar properties and would meet our needs. In the end, it is not any particular programming language that matters, but the ability to think with precision about both mathematical facts and computer programs.

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[1] Ronald L. Graham, Donald E. Knuth, Oren Patashnik, *Concrete Mathematics: A Foundation for Computer Science*, Second Edition, Addison-Wesley, 1998.

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We are very grateful to Dr. J. Adrian Zimmer of the Oklahoma School of Science and Mathematics for sharing his ideas about teaching math and Python. Adrian read a draft of this book and made valuable suggestions and corrections.

We thank Kenneth S. Oliver (formerly of Amity Regional High School in Woodbridge, Connecticut) for taking the time to read a draft very thoroughly, pointing out mistakes, and helping to clarify some explanations.

We are very grateful to Prof. Duncan A. Buell, Chair of the Department of Computer Science and Engineering at University of South Carolina in Columbia, for reading a draft and suggesting improvements and corrections, especially for the Number Theory and Cryptology chapter (Chapter 17).

Our thanks to Benjamin Niedzielski, Phillips Academy '08, for writing solutions to many exercises.

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In the second edition, Python code in all examples and exercises has been converted to Python 3. According to Python developers,

> Python 3.0, also known as 'Python 3000' or 'Py3K', is the first ever intentionally backwards-incompatible Python release. There are more changes than in a typical release, and more that are important for all Python users. Nevertheless, after digesting the changes, you'll find that Python really hasn't changed all that much – by and large, we're mostly fixing well-known annoyances and warts, and removing a lot of old cruft.[*]

We have inserted the "Parity, Invariants, and Finite Strategy Games" chapter after Chapter 9 and split the Graphs chapter into two, adding sections on graph coloring and the Four Color Theorem.

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[*] See http://docs.python.org/3.1/whatsnew/3.0.html.

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