Third Edition

Be Prepared

for the

AP

Calculus Exam

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Chapter 1. Exam Format, Grading, and Tips

1.1. Exam Format

Figure 1-1 shows the format of the AP Calculus exam. The exam takes 3 hours and 15 minutes of test time (plus breaks and time for instructions). It is divided into two sections. Section I consists of multiple-choice questions with a total allotted time of 1 hour and 45 minutes. Section II consists of free-response questions with a total allotted time of 1 hour and 30 minutes.

Each section consists of two parts: one with no calculator, the other with a calculator. In Section I, Part A contains 30 questions for 60 minutes; a calculator is not allowed on these questions. Part B contains 15 questions for 45 minutes; some of these questions require a graphing calculator. Section II starts with Part A, the calculator part: two questions for 30 minutes. For Part B, you have to put away your calculator. There are four questions for 60 minutes. During the Part B time period, you are allowed to go back to the two Part A free-response questions and continue working on them without your calculator.

The AB and BC practice exams in this book will give you a pretty accurate idea of what the questions are like. The multiple-choice questions cover a wide range of topics, including questions on definitions and fundamental concepts related to limits, continuity, derivatives, and integrals, as well as questions on common methods, rules, and formulas applied to finding limits, derivatives and antiderivatives, analysis of function graphs, related rates, linear approximation, distance-velocity-acceleration problems, areas of regions, volumes of solids, and so on. Both the AB and BC exams also include questions on separable differential equations and slope fields. ⌈ The BC exam adds questions on antidifferentiation by parts and by partial fractions, improper integrals, arc length, Euler's method, the logistic model, polar and parametric curves, series and Taylor and Maclaurin polynomials. ⌋

The free-response questions may include some theoretical elements but are mostly application problems. Each question consists of several parts (usually between 2 and 4); later parts may ask you to use the results from the previous parts.

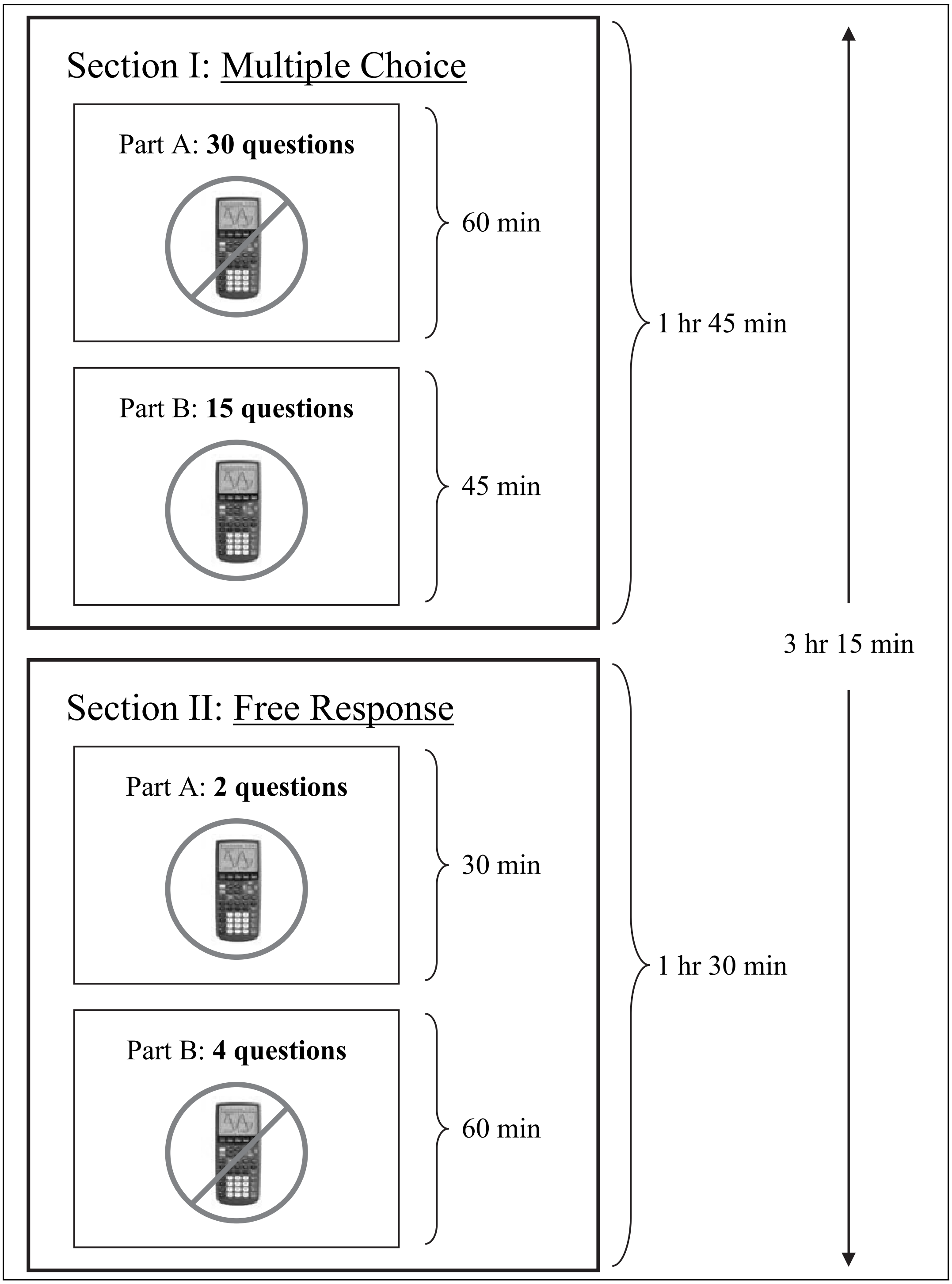

Figure 1-1. AP Calculus exam format

It's worth paying attention to recent exam trends.

Be Prepared "AP Calculus notes" link at `http://www.skylit.com/calculus` takes you to a web page that includes notes on recent changes in content or emphasis. For example, starting May 2011, points are no longer deducted for incorrect answers to multiple-choice questions.

Chapter 10 at `http://www.skylit.com/calculus` presents annotated solutions to recent free-response questions (and links to the questions). In recent years, the following types of questions have appeared with some regularity:

1. Questions about data presented in a table and/or a graph.
2. "Application of the integral" questions in which you are given a rate of change and need to find net change.
3. Questions in which you have to interpret the meaning of a derivative or integral in a particular context.
4. Questions that require you to justify the location of an extremum (either global or local) or a point of inflection on a graph. You need to know how to write such a justification.
5. Questions about $g(x)=\int_a^x f(t)\,dt$ for a specified function *f*, described by a graph or a formula.

1.2. Grading

The exams are graded on a scale from 1 to 5. Grades of 5 and 4 are called "extremely well qualified" and "well qualified," respectively, and usually will be honored by colleges that give credit or placement for AP exams in calculus. A grade of 3, "qualified," especially on the AB exam, may be denied credit or placement at some colleges. Grades of 2, "possibly qualified," and 1, "no recommendation," are very unlikely to earn you credit or placement.

Table 1-1 presents published statistics and grade distributions on the 2015 AB and BC exams. In 2015, 302,532 candidates took the AB exam, and 118,707 candidates took the BC exam.

The multiple-choice and free-response sections weigh equally in the final grade.

	Calculus AB		Calculus BC	
	Number	%	Number	%
Students	302,532	100.0	118,707	100.0
Grade:				
5	66,045	21.8	53,836	45.3
4	51,518	17.0	19,431	16.4
3	56,148	18.6	21,338	18.0
2	31,229	10.3	6,487	5.5
1	97,592	32.3	17,615	14.8
3 or Higher	173,711	57.4	94,605	79.7

Table 1-1. 2015 grade distributions for AB and BC exams*

To determine the grade, the College Board first calculates the total exam score. A weighted combination of the multiple-choice and free-response scores is used to determine the final total score:

Total score = MC coeff · correct count + FR coeff · FR score

For multiple-choice questions, one point is given for each correct answer.

Beginning with the 2011 exam, there is no deduction for wrong answers on multiple-choice questions.

Free-response questions are graded by a large invited group of high school teachers and college professors. Scores are assigned based on a *rubric* established by the Chief Reader, the Question Leader, and a group of exam readers. Each free-response question is scored out of 9 points, with partial credit given according to the rubric. The final score is obtained by adding the MC and FR weighted scores. The MC and FR coefficients are chosen in such a way that they give equal weights to the multiple-choice and free-response sections of the exam. For example, if the exam has 45 multiple-choice questions and 6 free-response questions, weights of 1.2 for multiple-choice and 1.0 for free-response will give each section a maximum total of 54, for a maximum possible total score of 108.

* Source: `http://research.collegeboard.org/programs/ap/data`.

	2003			2008	
	AB	BC		AB*	BC
AP Grade	1.2 * MC + 1.0 * FR		AP Grade	1.2272 * MC + 1.0 * FR	1.2 * MC + 1.0 * FR
5	66 - 108	64 - 108	5	65 - 108	66 - 108
4	47 - 65	53 - 63	4	48 - 64	55 - 65
3	29 - 46	36 - 52	3	34 - 47	39 - 54
2	16 - 28	26 - 35	2	21 - 33	31 - 38
1	0 - 15	0 - 25	1	0 - 20	0 - 30

Table 1-2. Score-to-grade conversion in 2003 and 2008

Four cut-off points determine the grade. Table 1-2 shows the actual cut-off points for 2003 and 2008 when this information was made public. In 2008, 60% or more correct answers on the AB exam and 61% or more correct answers on the BC exam would get you a 5. The cut-off points are determined by ETS statisticians and the Chief Reader and vary slightly from year to year based on the score distributions, equalization from year to year, and close examination of a sample of individual exams.

Before 2011, 1/4 point was deducted for each incorrect answer to an MC question. Without this deduction, the cut-off points would be higher. The College Board came up with estimates of what they would be in 2003 and 2008 (Table 1-3).

	2003			2008	
	AB	BC		AB	BC
AP Grade	1.2 * MC + 1.0 * FR		AP Grade	1.2272 * MC + 1.0 * FR	1.2 * MC + 1.0 * FR
5	69 - 108	67 - 108	5	68 - 108	69 - 108
4	51 - 68	57 - 66	4	52 - 67	59 - 68
3	35 - 50	41 - 56	3	39 - 51	44 - 58
2	23 - 34	31 - 40	2	27 - 38	36 - 43
1	0 - 22	0 - 30	1	0 - 26	0 - 35

Table 1-3. Estimated cut-offs in 2003 and 2008 if there were no deduction for incorrect answers to MC questions

* 2008 AB Question #19 was not counted because it "tricked" a disproportionate number of students.

1.3. AB or BC?

Table 1-1 shows that a larger percentage of BC exam takers got a 5. That's how it should be — if you have a choice and you haven't covered all the BC material, consider taking the AB exam. On the other hand, if you have taken a complete BC course, you should take the BC exam.

> **Every student who takes the BC exam receives a BC grade and also an AB subscore based on the questions that cover AB topics.**

The practice exams in this book will help you make up your mind about which exam to take.

Most colleges will take your AP courses and exam grades, if you take your exams early enough, into account in admissions decisions. But acceptance of AP exam results for credit and/or placement varies widely among colleges. In general, the AB exam is designed to correspond to a one-semester introductory course. The BC exam is designed to correspond to two one-semester courses. Some colleges give one-semester credit for the AB exam and two-semester credit for the BC exam, as intended. But other colleges may only give one semester of credit, regardless of the exam. They may also base their decision on your grade. For example, you may get a full year of credit only if you get a 5 on the BC exam. Some colleges may not give any credit at all. The College Board has collected links to AP acceptance policy statements at many colleges at `www.collegeboard.com/ap/creditpolicy/`.

To do well on the BC exam, you have to be comfortable with antidifferentiation by parts and by partial fractions, improper integrals, Euler's method, the logistic model, arc length, polar and parametric curves, and Taylor polynomials and series.

> **If you know this material, you shouldn't be afraid of the BC exam. Don't assume that the questions are "just harder." The BC exam simply includes more topics. You are not expected to solve problems any faster.**

Once you learn the additional BC topics, the BC exam questions are not necessarily harder than the AB exam questions. In fact, the AB and BC exams usually share three free-response and many multiple-choice questions. BC exam questions have to be more diverse in order to cover all the material in the same number of questions; this may actually make the exam easier for you if you have studied all the BC material. For example, you may understand series very well, but be prone to mistakes in questions that involve analysis of function graphs.

1.4. The Use of Graphing Calculators

You must bring a graphing calculator of an allowed model with you when you take the AP Calculus exam.

Not all calculators are allowed. In particular, non-graphing scientific calculators and calculators with a QWERTY keyboard are not allowed

Consult the list of allowed graphing calculator models at `https://apstudent.collegeboard.org/apcourse/ap-calculus-ab/calculator-policy`.

Laptops, tablets, any mobile phones and smart watches are not allowed.

You may bring two graphing calculators, if you wish, in case one of them stops working or if you have two different models that you commonly use.

You will use your calculator during Part B of Section I (multiple-choice) and Part A of Section II (free-response) only. You will be required to put it away during the remainder of the exam.

You are allowed to come in with programs or other information stored in your calculator. You will not be required to clear the memory prior to the exam.

The capabilities of different calculator models vary widely. Some of them (e.g., TI-89 and several of the HP calculators) have a built-in Computer Algebra System (CAS), which supports symbolic manipulation of expressions, such as symbolic differentiation. To ensure a level playing field, the Test Development Committee has defined four calculator operations that are sufficient to answer all AP exam questions:

1. produce a graph of a function within an arbitrary viewing window;
2. find the zeros of a function (that is, solve an equation numerically);
3. calculate the derivative of a function at a given value;
4. calculate the value of a definite integral.

> **It is essential to understand the rules of the game clearly. You are allowed to use your calculator in any way you want. At the same time, when writing your solutions to free-response questions and justifying your answers, you may refer only to the above four calculator operations.**

The above rules may sometimes make your life easier and at other times a little harder. On one hand, you do not have to be familiar with all the features of your calculator. If you know how and when to use the above four operations, you can do well on the exam. On the other hand, you have to watch out for inappropriate references to the calculator in your solutions.

> **If you give an answer in decimal approximation, it should be correct to three places after the decimal point, unless the question explicitly specifies other precision.**

1

Find the relative minimum of $f(x)=x^3-\sin x$. Justify your answer.

Enter

```
Y₁=X³-sin(X)
```

Plot it and you will see that there is a minimum at around $x=0.5$. So far so good. Now you might use the `minimum` command and find $x\approx 0.535$. Unfortunately, you won't receive credit if you simply write "using the minimum command we find $x\approx 0.535$," because "find a minimum" is not one of the four allowed calculator operations. Second attempt: you might enter

```
Y₁=X³-sin(X)
Y₂=nDeriv(Y₁(X),X,X)
```

plot `Y₂`, and find its zero at $x\approx 0.535$. Again, this solution won't get full credit because plotting a derivative is not one of the four allowed operations. You first need to honestly differentiate $f(x)=x^3-\sin x$. A correct solution might read like this:

> $f(x)=x^3-\sin x \Rightarrow f'(x)=3x^2-\cos x$. Using a calculator graph of $f'(x)$, we find that $f'(x)=0$ at $x\approx 0.535$ and $f'(x)$ changes sign from negative to positive there, so *f* has a relative minimum at $x\approx 0.535$.

The above example is slightly exaggerated. The people who write AP exam questions do not try to catch you on inappropriate calculator use — they make sure the question calls for a meaningful and unambiguous use of a calculator. Here you would have an advantage if you had a CAS calculator, because it can give you a formula for the derivative. So in reality a question like this would not appear on the open calculator part of the exam. Still, you should learn to play the calculator game by the rules.
To summarize, your familiarity with calculator features can be rather limited, but an excellent command of the four basic operations is essential, and you must understand the rules. The necessary calculator skills and examples are reviewed in this book's Calculator Skills appendix.

1.5. Exam-Taking Tips

The very first step to doing well on the exam is to make a personal commitment to do the very best you can. You've already taken the next step: you've picked up this book to prepare. Now give yourself ample review time. Hopefully, you haven't waited until the last week to start your studying. What you are seeking is <u>comfort</u> with the material; you want to feel secure that you <u>know calculus</u> as you enter that exam room. This book, besides breaking up the material into manageable chunks and organizing it for you, should help you focus on main ideas, eliminating the need for rote memorization.

The following are some tips for the multiple-choice portion of the exam:

1. Answer every question. There is no penalty for a wrong answer, so if you find you are running out of time and have left some questions unanswered, you should guess.
2. If a common paragraph, graph, or table refers to a group of questions and you took the time to read it, try each question in the group.
3. Don't go back and change an answer unless you have found an error in your work. Your first impulse is more likely to be correct.
4. Some multiple choice questions can be answered by working backwards. Plug the answer choices into the problem and see which answer works out.
5. Don't get stuck. Skip a hard question and come back to it if you have time after finishing the rest. But be careful marking your answer sheet when you skip questions.
6. On the calculator portion, you won't need your calculator on all the questions. If all or some of the answers have three digits after the decimal point (e.g., 98.765), it's time to pick up the calculator (but, of course, there may be other times to use it as well).

On the free-response section, try to write clearly, be neat, and keep your exam reader in mind. State the answer clearly, perhaps circle it.

Remember that all six free-response questions have equal weight. Don't assume that the first question is the easiest and the last is the hardest. Be sure to try all the questions, including Question 6.

Other things to remember about free-response questions:

1. Do read the question before jumping to the equations, formulas, or graphs included in the question.
2. Don't waste your time erasing large portions of work. Instead, cross out your work with one neat line, but only after you have something better to replace it with.
3. On the calculator portion, read through the two questions and begin with the ones that require the use of a calculator. After 30 minutes, you'll have to put your calculator away, but you'll be able to keep working on the questions. So save for last those questions where you don't need the calculator.
4. Show your work. Explain your answer in a sentence or two if the question requires it.
5. If a problem names a function, like $f(x)=\sqrt{x}+\sin\left(\frac{x^3}{37}\right)$, use the name of the function in your work instead of copying the actual function definition. This reduces the chance of making a transcription error, and saves time.
6. Store intermediate results on your calculator to be used later in the problem (see Appendix: Calculator Skills). Don't do any rounding until the end of the problem.
7. Write down in the exam book any integral or derivative you evaluate or any equation you solve on your calculator.
8. Try every part of every question. Even if you cannot complete Part (a) of a question, you may still be able to do the remaining parts.
9. Don't give a "recipe" for your answer. Just do the work. Don't waste time announcing how you'll proceed before doing the problem.
10. Be sure to answer the question, but don't do excessive writing. For example, if a question asks when a particle is moving to the right, be sure to answer it, but don't also tell when it is moving to the left.

11. Don't waste time simplifying answers, especially with derivatives and linear equations. You will lose a point if you simplify incorrectly. An answer like $\sin\left(\frac{\pi}{3}\right)-2\sin\left(\frac{\pi}{6}\right)$ is just as good as $\frac{\sqrt{3}}{2}-1$. A tangent line written in the form $y-3=\frac{5}{3}(x-6)$ is just as good as $y=\frac{5}{3}x-7$.

12. Watch for the phrases like "Justify your answer" or "Explain your reasoning." They mean you must have your work shown and write a sentence or two explaining your answer. A picture or a graph alone is never a justification.

13. Try not to use the pronoun "it." The phrase "it is increasing" could refer back to a given function, its derivative, its antiderivative, or other things. If a reader doesn't know to which function the word "it" refers, the reader will not be able to award credit. Be specific in your explanations.

14. Don't quit until the time is up. Use all the time you have and keep trying. The test will be over before you know it.