Third Edition

Be Prepared



Calculus Exam

Mark Howell Gonzaga High School, Washington, D.C.

Martha Montgomery Fremont City Schools, Fremont, Ohio

> Appendix: Calculator Skills (TI-89)

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Skylight Publishing 9 Bartlet Street, Suite 70 Andover, MA 01810

web: <u>http://www.skylit.com</u> e-mail: sales@skylit.com support@skylit.com

Appendix: Calculator Skills (TI-89)

The Test Development Committee has defined four calculator operations that are sufficient to answer all AP exam questions:

- 1. produce a graph of a function within an arbitrary viewing window;
- 2. find the zeros of a function (i.e., solve an equation numerically);
- 3. calculate the derivative of a function at a given value;
- 4. calculate the value of a definite integral.

You should practice these skills prior to the AP Exam. A few examples follow, with calculator-assisted solutions for the TI-89. The TI-83 / TI-84, TI-Nspire, and HP Prime models are described in separate documents. There are other acceptable calculator methods to solve these problems. If your calculator model does not match one of the models presented, consult your user's manual to solve the examples.

A.1. Graphing a Function

This is the simplest calculator skill required on the exam. Usually, the hardest part is making sure you enter the function correctly on your calculator, and that you choose a suitable viewing window. Be sure to check that the parentheses that enclose function arguments (as in sin(X)) are properly matched.

Be sure that your calculator is set to the Radian mode when you take the exam. (To set the mode, go to the MODE menu.)

You will see several graphing examples in the following sections.

A.2. Solving an Equation

Graphing calculators offer several methods for solving an equation.

The easiest approach is to enter the functions for each side of the equation, graph the two functions, and use the Intersection command in the MATH menu on the GRAPH screen. However, this method may <u>not</u> give the required accuracy.

If you are looking for a root of an equation or a zero of a function, graph the function and use the Zero command to find the *x*-intercept of the graph.

Example 1

The derivative of a function *f* is given by $f'(x) = \sin(\sqrt{x^2 + 1}) - \cos(\sqrt{x})$. Find all the values of *x* in the open interval (0, 6) where *f* has a local minimum.

Solution

We need to determine where the derivative changes sign from negative to positive. It is a good idea to graph the given derivative first, especially when you are given a domain of values for the independent variable. Then use the Root command from the MATH menu. Here are the necessary steps.

First press WINDOW to set your viewing window:

(F1+) F2+ Tools200m xmin=0. xmax=6. xscl=1. ymin=-1. ymax=1. yscl=1. xres=1		
MAIN	RAD AUTO	FUNC

Note that the choices of xmin and xmax correspond to the interval given in the problem. Press y= and set y_1 to f'(x), taking care that your parentheses are properly matched. Then press GRAPH to plot the graph:



You can see two points where the derivative changes sign from negative to positive. Go to the MATH menu (F5), and choose 2:Zero:



You will be prompted to enter a left bound for the zero. You can either use the arrow keys to move the cursor to the left of the leftmost zero, or simply enter a number for x using the keypad. You have to be sure the number is to the left of the zero you are looking for (but still in the viewing window). Type in 0 for x:



Then either move the cursor over to the right of the zero, or enter a value of x that you know is to the right of the zero (but not past the next zero). Suppose you use the cursor; press ENTER when you are satisfied with its position:



The zero will then be calculated, and stored into the variable xc:



Repeat the process to find the other zero where the derivative changes sign from negative to positive:



The answers (accurate to three digits to the right of the decimal) are $x \approx 0.283$ and $x \approx 5.432$.

For the exam, you must use the MATH/root or MATH/Intersection command from the MATH menu on the Graph screen, (or the nsolve () or solve () command from the HOME screen discussed below) to find a root or an intersection point. Just tracing along a graph to find roots or intersection points may not give you the required accuracy.

Example 2

The number of bees in a colony is given by $B(t) = 523e^{0.18t}$, where *t* is the number of days since the colony was established. The derivative of B(t) is given by $B'(t) = 94.14e^{0.18t}$. On what day is the number of bees in the colony increasing at the rate of 1000 bees per day?

Solution

On the HOME screen, press F2 Algebra, then 8:nSolve to bring up the numeric solver. We are using the nSolve command for this problem because we have no way of knowing what window settings we might need to use the graph environment. Enter the equation $94.14e^{0.18t} = 1000$, a comma, and the variable *t*, as shown:



Be patient: this operation may take a few seconds.

 $t \approx 13.128$; the answer is on the 14th day.

The nSolve command uses numerical methods to find one solution to the equation you specify. If there is more than one solution, it usually will find the solution nearest the value currently stored in the variable being solved for. You can also specify a close guess in the command. For example, if you enter nSolve(sin(x) = 0.5, x = 3), the TI-89 will return 2.617994 as the result, the solution closest to x = 3.

You can also use the solve command to solve an equation. It tries to give all the solutions to the equation or all the solutions within an interval that you specify. For example, the command solve $(sin(x)=0.5, x=3) \mid x>0$ and $x<2\pi$ gives the results x = 2.617994 and x = 0.523599.

Example 3

The derivative of a function g is given by $g'(x) = \frac{x}{2} - \cos(x^2) + 0.3$. What is the x-coordinate of a local maximum point on the graph of g?

Solution

Enter g'(x) into y1, press F2 and select 4 to graph it in the ZoomDec window:





We need to find a zero where g'(x) changes sign from positive to negative. From the graph, this zero appears to be near x = -1. We could use the zero command on the graph to find the answer, as shown in Example 1. Here, we will use the numeric solver app. Press APPS and select 9:NumericSolver, enter the equation as shown, then enter a "seed" value of -1 for x:

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MAIN	RAD AUTO FUNC			MAIN BAD AUTO FUNC

Position the cursor on the x = line and press F2 to solve for x:

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	!={-1.е14; rt=7.е-1;		
-10.0			
MAIN	RAD AUTO	FUNC	

The local maximum occurs at the point where $x \approx -1.410$.

A.3. Evaluating a Derivative at a Point

Example 4

Find the slope of the line tangent to the graph of $y = 2x + \sin(1 + x^2) + \cos(1 - x^2)$ at the point where the graph crosses the *x*-axis.

Solution

First enter the function into y1:



Then use the procedure from the previous section to find the zero of a function:



To evaluate a derivative, go to the HOME screen, press F3, and select 1:d(differentiate. You could also enter the derivative command directly by pressing 2nd 8. Enter the arguments shown on the edit line below. We are evaluating the derivative at the value of xc, which is the *x*-coordinate of the cursor on the graph screen.

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This is the zero of the function $y = 2x + \sin(1 + x^2) + \cos(1 - x^2)$.

A.4. Evaluating an Integral Numerically

On the open-calculator free-response part of the AP exam (Section II, Part A), always use your calculator when you need to evaluate a definite integral. You don't get "extra credit" for evaluating an integral by first finding an antiderivative.

Example 5

Find the area of the region in the first quadrant bounded by the graphs of $f(x) = \sin(x^2) + \cos(x^2) + x^2$, $g(x) = e^x - 1$, and the *y*-axis.

Solution

It is a good idea to clear out all your single letter variables before starting a problem like this. You do that by pressing 2nd F1 ENTER Clear a-z:



To find the area, we must first find where the curves intersect. Enter f(x) as y1 and g(x) as y2. Press F2 and select 4 to graph in the ZoomDec window. The screen below shows where to find the Intersection command:

F1+ F2+ Tools Zoom	F3 F4 F5+ F6+ F7+ ⁶ !! TraceRe9raph <mark>Math</mark> DrawPen>:
	1:Value 2:Zero
	3 Minimum 4 Maximum
	5:Intersection 6:Derivatives >
	7:Jf(x)dx 8↓Inflection
MAIN	RAD AUTO FUNC

You will be asked to identify the first curve and the second curve, which you can do by pressing ENTER. Then you need to choose lower and upper bounds for the *x*-coordinate of the point of intersection. Follow the procedure outlined in Example 1.

After you've found the intersection, go to the HOME screen and store the current value of xc into the variable a. The STO> key is located directly above the ON key.

Now you are ready to evaluate the integral and get the area: Area = $\int_{0}^{a} y l(x) - y 2(x) dx$. To evaluate a definite integral, on the HOME screen, press

F3 and select 2: \int (<code>integrate.</code> Or use the integral key found on the keypad,

2nd 7. Enter the arguments as shown in the screen below:

F1+ F2+ ToolsAl9ebro	F3+ F4+ aCalcOther	F5 Pr9mi0	F6+ Clean Up	\square
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Press \blacklozenge ENTER after typing the integral to force the calculator to give you an approximate result.