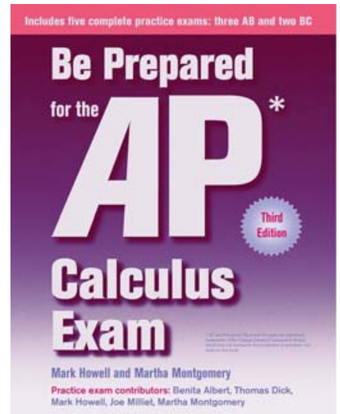


# Be Prepared
for the
# AP
# Calculus Exam

Third Edition

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**Chapter 10. Annotated Solutions to Past Free-Response Questions**

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# 2016 AB

## AP Calculus Free-Response Solutions and Notes

**Question AB-1**

(a) $R'(2) \approx \dfrac{R(3)-R(1)}{3-1} = \dfrac{950-1190}{3-1} = -120$ liters per hour per hour.

(b) The left Riemann sum approximation is $1340 \cdot 1 + 1190 \cdot 2 + 950 \cdot 3 + 740 \cdot 2 = 8050$ liters. This is an overestimate, since $R(t)$ is decreasing for $0 \le t \le 8$.

(c) The amount of water pumped into the tank from $t = 0$ to $t = 8$ is $\int_0^8 W(t)\,dt \approx 7836.195$. [1] So the amount of water in the tank at $t = 8$ is approximately $50000 + 7836.195 - 8050 = 49786$ liters.

(d) Yes. The function $f(t) = W(t) - R(t)$ is continuous. $f(0) = 2000 - 1340 > 0$ and $f(8) \approx 81.524 - 700 < 0$. By the Intermediate Value Theorem, there is a time *t*, $0 < t < 8$ when $f(t) = 0$. At that time, $W(t) = R(t)$.

**Notes:**

1. Use the given names for the functions in your answers, rather than their formulas, to avoid transcription errors.

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**Question AB-2**

(a) $v(4) \approx 2.979$ and $v'(4) \approx -1.164$. Since the velocity is positive and decreasing at $t = 4$, the particle is slowing down.

(b) The particle changes direction only at $t \approx 2.707$ on the interval $0 < t < 3$, since the velocity changes sign at that time and at no other time on that interval.

(c) $x(0) = 2 + \int_4^0 v(t)\,dt \approx -3.815$.

(d) $\int_0^3 |v(t)|\,dt \approx 5.301$.

---

**Question AB-3**

(a) Neither. $g'(x) = f(x)$ and $f(x)$ does not change sign at $x = 10$.

(b) Yes, because $g'(x) = f(x)$ changes from increasing to decreasing at $x = 4$.

(c) The only points where $g$ could have an absolute maximum or absolute minimum are $x = -4$, $x = -2$, $x = 6$, and $x = 12$. $g(-4) = -4$, $g(-2) = -8$, $g(6) = 8$, $g(12) = -4$. The absolute minimum is $-8$ and the absolute maximum is 8. [1]

(d) $g(x) \le 0$ on $-4 \le x \le 2$ and on $10 \le x \le 12$. [2]

**Notes:**

1. The candidate test is the easiest way to justify absolute extrema for a continuous function on a closed interval.

2. $g(2) = 0$ and for $-2 < x < 2$ $g(x) < 0$, because the integrand is positive and we are integrating from right to left. Integrating from $x = -2$ to $x = -4$ only increases $g$ by 4, not enough to make it positive. For $x > 2$, $g(10)$, and $g$ decreases on the interval [10, 12].

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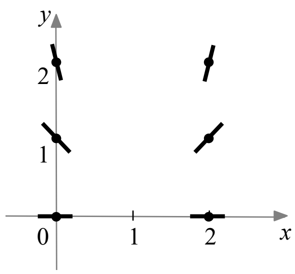
**Question AB-4**

(a)




(b) $\left.\dfrac{dy}{dx}\right|_{x=2,\ y=3} = 9$. An equation for the tangent line is $y-3=9(x-2)$.
$f(2.1) \approx 3 + 9 \cdot 0.1 = 3.9$. [1]

(c) Separating variables, we get $y^{-2}dy = \dfrac{dx}{x-1} \Rightarrow \int y^{-2}dy = \int \dfrac{dx}{x-1} \Rightarrow$
$-\dfrac{1}{y} = \ln|x-1| + C$. Using the initial condition, $-\dfrac{1}{3} = \ln(1) + C \Rightarrow C = -\dfrac{1}{3}$. So
$-\dfrac{1}{y} = \ln|x-1| - \dfrac{1}{3} \Rightarrow \dfrac{1}{y} = \dfrac{1}{3} - \ln|x-1|$. Since the initial condition is given at $x = 2$,
we can remove the absolute value: $f(x) = \dfrac{1}{\dfrac{1}{3} - \ln(x-1)}$. [2]

**Notes:**

1. Be careful not to make the claim that $f(2.1) = 3.9$. The use of = in approximations of this kind is usually penalized.

2. The natural domain of this solution is $1 < x < 1 + \sqrt[3]{e}$, but it is not necessary to include the domain unless it is specifically asked for in the problem.

**Question AB-5**

(a) $\frac{1}{10}\int_0^{10}\left(\frac{1}{20}\left(3+h^2\right)\right)dh=\frac{1}{200}\int_0^{10}\left(3+h^2\right)dh=\frac{1}{200}\left(3h+\frac{h^3}{3}\right)\Bigg|_0^{10}=$

$\frac{1}{200}\left(30+\frac{1000}{3}\right)$ inches. [1]

(b) Volume = $\int_0^{10}\pi r^2 dh=\pi\int_0^{10}\left(\frac{1}{20}\left(3+h^2\right)\right)^2 dh=\frac{\pi}{400}\int_0^{10}\left(9+6h^2+h^4\right)dh=$ .

$\frac{\pi}{400}\left(9h+2h^3+\frac{h^5}{5}\right)\Bigg|_0^{10}=\frac{\pi}{400}\left(90+2000+20000\right)$ cubic inches. [2]

(c) $\frac{dr}{dt}=\frac{1}{20}\left(2h\frac{dh}{dt}\right)$. At $h = 3$, this is

$\frac{1}{20}\left(2h\frac{dh}{dt}\right)=\frac{1}{20}\cdot 2\cdot 3\cdot\frac{dh}{dt}=-\frac{1}{5}\ \Rightarrow\ \frac{dh}{dt}=-\frac{2}{3}$ inches per second.

**Notes:**

1. You can leave the answer unsimplified. For the record, it is $\frac{109}{60}$.

2. Again, the unsimplified answer is sufficient to receive full credit. This is $\frac{2209\pi}{40}$.

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**Question AB-6**

(a) $k(3)=f(g(3))=f(6)=4$ and $k'(3)=f'(g(3))\cdot g'(3)=f'(6)\cdot g'(3)=5\cdot 2=10$.
An equation for the tangent line is $y-4=10(x-3)$.

(b) $h'(1)=\frac{f(1)g'(1)-g(1)f'(1)}{f^2(1)}=\frac{-48-6}{36}=-\frac{3}{2}$.

(c) $\int_1^3 f''(2x)\,dx=\frac{1}{2}f'(2x)\Big|_1^3=\frac{1}{2}\left(f'(6)-f'(2)\right)=\frac{1}{2}\left(5-(-2)\right)=\frac{7}{2}$.

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# 2016 BC
## AP Calculus Free-Response Solutions and Notes

**Question BC-1**

See AB Question 1.

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**Question BC-2**

(a) $x(3) = x(0) + \int_0^3 x'(t)\,dt = 5 + \int_0^3 t^2 + \sin\left(3t^2\right) dt \approx 14.377$. From the graph, $y(3) = -\frac{1}{2}$. At $t = 3$. The particle is at $\left(14.377,\ -\frac{1}{2}\right)$.

(b) The slope is $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}\Big|_{t=3} = \frac{1/2}{9+\sin(27)} \approx 0.050$.

(c) The speed is $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(9+\sin(27)\right)^2 + \left(\frac{1}{2}\right)^2} \approx 9.969$. [1]

(d) Distance traveled =

$$\int_0^1 \sqrt{\left(t^2+\sin\left(3t^2\right)\right)^2 + (-2)^2}\,dt + \int_1^2 \sqrt{\left(t^2+\sin\left(3t^2\right)\right)^2 + (0)^2}\,dt \approx 4.350.$$ [2]

**Notes:**

1. Or 9.968, if truncated at the third decimal place.

2. Or 4.349, if truncated at the third decimal place.

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**Question BC-3**

See AB Question 3.

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**Question BC-4**

(a) $\frac{d^2y}{dx^2} = 2x - \frac{1}{2}\frac{dy}{dx} = 2x - \frac{1}{2}\left(x^2 - \frac{1}{2}y\right) = 2x - \frac{x^2}{2} + \frac{y}{4}$.

(b) At $(-2, 8)$, $\frac{dy}{dx} = (-2)^2 - \frac{8}{2} = 0$ and $\frac{d^2y}{dx^2} = 2(-2) - \frac{4}{2} + \frac{8}{4} = -4 < 0$, so the particular solution has a local maximum at $(-2, 8)$.

(c) Both the numerator and the denominator approach 0, so we can apply L'Hospital's Rule: $\lim_{x\to -1}\left(\frac{g(x)-2}{3(x+1)^2}\right) = \lim_{x\to -1}\left(\frac{g'(x)}{6(x+1)}\right)$. Again, both the numerator and the denominator approach 0, so we can apply L'Hospital's Rule a second time: $\lim_{x\to -1}\left(\frac{g'(x)}{6(x+1)}\right) = \lim_{x\to -1}\left(\frac{g''(x)}{6}\right)$. From Part (a), $g''(-1) = 2(-1) - \frac{1}{2} + \frac{1}{2} = -2 \Rightarrow$ $\lim_{x\to -1}\left(\frac{g''(x)}{6}\right) = \frac{-2}{6} = -\frac{1}{3}$.

(d) At $(0, 2)$ the slope is $-1$. $y(0.5) \approx 2 - 1\cdot\frac{1}{2} = \frac{3}{2}$. At $\left(\frac{1}{2}, \frac{3}{2}\right)$ the slope is $\frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$. $y(1) \approx \frac{3}{2} - \frac{1}{2}\cdot\frac{1}{2} = \frac{5}{4}$.

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**Question BC-5**

See AB Question 5.

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**Question BC-6**

(a) $1-\frac{1}{2}(x-1)+\frac{1}{4}\frac{(x-1)^2}{2!}-\frac{2}{8}\frac{(x-1)^3}{3!}+\ldots+\frac{(-1)^n(n-1)!}{2^n n!}(x-1)^n+\ldots=$

$1-\frac{1}{2}(x-1)+\frac{1}{8}(x-1)^2-\frac{1}{24}(x-1)^3+\ldots+\frac{(-1)^n}{n\cdot 2^n}(x-1)^n+\ldots$.

(b) At $x=-1$, the series is

$1-\frac{1}{2}(-2)+\frac{1}{8}(-2)^2-\frac{1}{24}(-2)^3+\ldots+\frac{(-1)^n}{n\cdot 2^n}(-2)^n+\ldots=1+1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$, which is the harmonic series with an additional term, so diverges. At $x=3$, the series is

$1-\frac{1}{2}(2)+\frac{1}{8}(2)^2-\frac{1}{24}(2)^3+\ldots+\frac{(-1)^n}{n\cdot 2^n}(2)^n+\ldots=1-1+\frac{1}{2}-\frac{1}{3}\ldots+\frac{(-1)^n}{n}+\ldots$, which converges, since it is the alternating harmonic series with an additional term. So the interval of convergence is $-1<x\le 3$.

(c) $f(1.2)\approx 1-\frac{1}{2}\cdot(0.2)+\frac{1}{8}(0.2)^2$. [1]

(d) The magnitude of the error is bounded by the magnitude of the first omitted term, which is $\left|-\frac{1}{24}(0.2)^3\right|=\frac{1}{3000}<\frac{1}{1000}=0.001$.

**Notes:**

1. Leave it at this to save time and avoid arithmetic mistakes. (This is equal to 0.905.)