Be Prepared

for the

AP

Calculus Exam

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Chapter 10.   Annotated Solutions to Past Free-Response Questions

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2011 AB

AP Calculus Free-Response
Solutions and Notes

Question AB-1

(a) $v(5.5) \approx -0.453$ and $a(5.5) \approx -1.359$. The velocity is decreasing because $v'(5.5) = a(5.5) < 0$. Since $v(5.5)$ and $a(5.5)$ have the same sign (both negative), the speed is increasing.

(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt \approx 1.949$.

(c) Total distance traveled $= \int_0^6 |v(t)|\,dt \approx 12.573$.

(d) $v(t)$ changes sign at $t \approx 5.19552$. Let b = 5.19552[1].

$$x(b) = x(0) + \int_0^b v(t)\,dt = 2 + \int_0^b v(t)\,dt \approx 14.135.$$

Notes:

1. Store this number in your calculator, and use the stored value in the subsequent calculation.

Question AB-2

(a) $H'(3.5) \approx \dfrac{H(5)-H(2)}{5-2} = \dfrac{52-60}{3} = -\dfrac{8}{3}$ degrees Celsius/minute.

(b) $\dfrac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea in degrees Celsius from $t = 0$ to $t = 10$ minutes.[1] The trapezoidal approximation gives
$\dfrac{1}{10}\left(2\cdot\dfrac{66+60}{2}+3\cdot\dfrac{60+52}{2}+4\cdot\dfrac{52+44}{2}+1\cdot\dfrac{44+43}{2}\right)$.[2]

(c) By the FTC, $\int_0^{10} H'(t)\,dt = H(10)-H(0) = 43-66 = -23$ degrees Celsius**.**

$\int_0^{10} H'(t)\,dt$ represents the net change in temperature of the tea, in degrees Celsius, over the 10-minute time interval.[3]

(d) The temperature of biscuits at $t = 10$ minutes
$B(10) = B(0)+\int_0^{10} B'(t)\,dt = 100+\int_0^{10} B'(t)\,dt$ 🖩 ≈ 34.183. The biscuits are
43 – 34.183 = 8.817 degrees Celsius cooler than the tea.

Notes:

1. It is important to mention the units both for the variable *t* (minutes) and the average temperature (degrees Celsius).
2. No need to evaluate. For the curious, the value is $\approx \dfrac{1}{10}(529.5) = 52.950$.
3. An alternative meaning might be "The tea cooled by 23°C during the 10 minutes," but that would describe the meaning of the <u>value</u> of the expression, while the problem clearly asks for the meaning of the <u>expression</u> itself.

Question AB-3 [1]

(a) $f\left(\frac{1}{2}\right)=1$. $f'(x)=24x^2$ so $f'\left(\frac{1}{2}\right)=6$. The tangent line is $y-1=6\cdot\left(x-\frac{1}{2}\right)$. [2]

(b) Area = $\int_0^{\frac{1}{2}}\left(\sin(\pi x)-8x^3\right)dx=\left(-\frac{1}{\pi}\cos(\pi x)-2x^4\right)\Bigg|_0^{\frac{1}{2}}=0-\frac{1}{8}-\left(-\frac{1}{\pi}\right)=\frac{1}{\pi}-\frac{1}{8}$.

(c) Volume = $\pi\int_0^{1/2}\left(\left(1-f(x)\right)^2-\left(1-g(x)\right)^2\right)dx$. [3]

Notes:

1. This is the third consecutive year that an area-volume problem has appeared in the <u>closed</u> calculator part of the free response section. This trend is likely to continue.
2. Or $y=1+6\cdot\left(x-\frac{1}{2}\right)$. Don't waste time to simplify.
3. Use the names of the functions given in the problem to save time and prevent transcription errors.

Question AB-4

(a) $g(-3) = -6 + \int_0^{-3} f(t)\,dt = -6 - \dfrac{9\pi}{4}$.[1] $g'(x) = 2 + f(x)$, so $g'(-3) = 2 + 0 = 2$.

(b) $g'(x)$ changes sign from positive to negative only once on $[-4, 3]$, so *g* has an absolute maximum when this happens. For $-4 \le x \le 0$, $g'(x) = 2 + f(x) > 0$. For $0 \le x \le 3$, $g'(x) = 0$ when $f(x) = -2$. $f(x) = 3 - 2x$, so $f(x) = -2$ when $x = \dfrac{5}{2}$. Since $g'(x) > 0$ for $-4 < x < \dfrac{5}{2}$ and $g'(x) < 0$ for $\dfrac{5}{2} < x < 3$, the absolute maximum is at $x = \dfrac{5}{2}$.[2]

(c) Only $x = 0$. $g'(x) = 2 + f(x)$ changes from increasing to decreasing only at $x = 0$, and $g'(x)$ never changes from decreasing to increasing. So the graph of *g* changes from concave up to concave down, at $x = 0$.

(d) The average rate of change is $\dfrac{f(3) - f(-4)}{3 - (-4)} = \dfrac{-3+1}{7} = -\dfrac{2}{7}$. There is no contradiction with the Mean Value Theorem because the condition that f is differentiable on the interval is not satisfied: *f* is not differentiable at $x = 0$.

Notes:

1. The integral from 0 to -3 is negative.
2. Since $g'(x)$ changes sign from positive to negative only once in the interval, it is easier to use this argument than the candidate test.

Question AB-5

(a) $\left.\dfrac{dW}{dt}\right|_{t=\frac{1}{4}} = \dfrac{1}{25}\cdot\left(1400-300\right) = 44$. The tangent line is $y = 1400 + 44t$. At $t = \dfrac{1}{4}$, $W \approx 1411$ tons.

(b) $\dfrac{d^2W}{dt^2} = \dfrac{1}{25}\cdot\left(\dfrac{dW}{dt}\right) = \dfrac{1}{25}\cdot\left(\dfrac{1}{25}\cdot\left(W-300\right)\right) = \dfrac{1}{625}\left(W-300\right)$. For $0 < t < \dfrac{1}{4}$, $W \ge 1400 > 300$, so $\dfrac{d^2W}{dt^2} > 0$. Therefore the tangent line lies under the graph of $y = W(t)$. Therefore, 1411, the estimate based on the tangent line, underestimates the actual value.

(c) $\displaystyle\int \frac{1}{W-300}\,dW = \int \frac{1}{25}\,dt \;\Rightarrow\; \ln\left|W-300\right| = \frac{1}{25}t + C$. Substituting $W = 1400$ at $t = 0$ we get $\ln\left(1100\right) = C$. Since $W(t)$ is increasing, $W - 300 > 0$. Solving for W, we get $W - 300 = e^{\frac{t}{25}+\ln(1100)} \;\Rightarrow\; W = 300 + e^{\frac{t}{25}+\ln(1100)}$ [1] or $W = 300 + 1100e^{\frac{t}{25}}$.

Notes:

1. No need to simplify further.

Question AB-6

(a) $f(x)$ is continuous when $x \neq 0$. $\lim\limits_{x\to 0^+} f(x) = 1$ and $\lim\limits_{x\to 0^-} f(x) = 1$, so $\lim\limits_{x\to 0} f(x) = 1 = f(0)$. Therefore, *f* is continuous at *x* = 0, too.

(b) $f'(x) = \begin{cases} -2\cos x, & \text{for } x < 0 \\ -4e^{-4x}, & \text{for } x > 0 \end{cases}$. $-4e^{-4x} = -3 \;\Rightarrow\; e^{-4x} = \dfrac{3}{4} \;\Rightarrow\; x = -\dfrac{1}{4}\ln\left(\dfrac{3}{4}\right)$.

(c) Average value =

$$\frac{1}{1-(-1)}\int_{-1}^{1} f(x)\,dx = \frac{1}{2}\left(\int_{-1}^{0}(1-2\sin x)\,dx + \int_{0}^{1} e^{-4x}dx\right) =$$

$$\frac{1}{2}\left(\left(x+2\cos x\right)\Big|_{-1}^{0} + \left(\frac{e^{-4x}}{-4}\right)\Bigg|_{0}^{1}\right) = \frac{1}{2}\left(\left(2-\left(-1+2\cos(-1)\right)\right) + \left(\frac{e^{-4}}{-4} - \frac{1}{-4}\right)\right).$$ [1]

Notes:

1. No need to simplify further. If you insist, $= \dfrac{13}{8} - \cos(-1) - \dfrac{1}{8e^4}$.

2011 BC
AP Calculus Free-Response
Solutions and Notes

Question BC-1

(a) Speed = $\left.\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\right|_{t=3} = \sqrt{13^2+\sin^2(9)} \approx 13.007$.

$\vec{a}(3) = \left.\left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\right)\right|_{t=3} = (4, 6\cos(9)) \approx (4, -5.467)$. [1]

(b) The slope is $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt}$. At t = 3, this is $\frac{\sin 9}{13} \approx 0.032$.

(c) $x(3) = x(0) + \int_0^3 \frac{dx}{dt}\,dt = \int_0^3 (4t+1)\,dt = 21$;

$y(3) = y(0) + \int_0^3 \frac{dy}{dt}\,dt = -4 + \int_0^3 \sin(t^2)\,dt \approx -3.226$.

(d) Total distance traveled is $\int_0^3 \sqrt{(4t+1)^2 + (\sin(t^2))^2}\,dt \approx 21.091$

Notes:

1. You could use numeric derivatives of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ instead.

Question BC-2

See AB Question 2.

Question BC-3 ▢[1]

(a) Perimeter = $1+k+e^{2k}+\int_0^k \sqrt{1+\left(2e^{2x}\right)^2}\,dx$

(b) Volume = $\pi\int_0^k \left(e^{2x}\right)^2 dx = \pi\int_0^k e^{4x}dx = \frac{\pi}{4}e^{4x}\Big|_0^k = \frac{\pi}{4}\left(e^{4k}-1\right)$.

(c) $\frac{dV}{dt} = \frac{\pi}{4}\cdot 4e^{4k}\frac{dk}{dt}$. At $k=\frac{1}{2}$, $\frac{dV}{dt} = \frac{\pi}{4}\cdot 4e^2\cdot\frac{1}{3} = \frac{\pi e^2}{3}$.

▢ **Notes:**

1. This is the third consecutive year that an area-volume problem has appeared in the closed calculator part of the free response section. This trend is likely to continue.

Question BC-4

See AB Question 4.

Question BC-5

See AB Question 5.

Question BC-6

(a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$. $\sin\left(x^2\right) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \ldots$

(b) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$.

$$f(x) = \left(x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \ldots\right) + \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\right) =$$

$$1 + \frac{x^2}{2} + \frac{x^4}{4!} - \left(\frac{1}{3!} + \frac{1}{6!}\right)x^6 + \ldots$$ 1

(c) $\frac{f^{(6)}(0)}{6!} = -\left(\frac{1}{3!} + \frac{1}{6!}\right) \Rightarrow f^{(6)}(0) = -\left(\frac{6!}{3!} + 1\right) = -121$.

(d) From the graph, the maximum value of $\left|f^{(5)}(x)\right|$ on the interval $\left(0, \frac{1}{4}\right)$ is less than 40. 2 The Lagrange error bound is less than

$\frac{40}{5!} \cdot \left(\frac{1}{4}\right)^5 = \frac{1}{3 \cdot 4^5} = \frac{1}{3 \cdot 1024} < \frac{1}{3000}$. So $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

Notes:

1. Leave it at that.
2. We chose 40 because undoubtedly $\left|f^{(5)}(x)\right| < 40$ on $\left(0, \frac{1}{4}\right)$, 40 is convenient to use in calculations, and 40 gives the desired error bound of less than $\frac{1}{3000}$. 35 would work as well, but the arithmetic would be slightly more difficult. The choice of 30 doesn't work because $\left|f^{(5)}\left(\frac{1}{4}\right)\right|$ is slightly greater than 30.

2011 AB (Form B)
AP Calculus Free-Response
Solutions and Notes

Question AB-1 (Form B)

(a) $S(0)=0$. $S(60)=S(0)+\int_0^{60} S'(t)\,dt \approx 171.813$ millimeters.

(b) The average rate of change is $\dfrac{S(60)-S(0)}{60-0} \approx \dfrac{171.813}{60} \approx 2.864$ millimeters/day.

(c) The volume at time *t*, $V(t)=\pi r^2 S(t)=100\pi S(t)$. The rate of change of the volume is $V'(t)=100\pi S'(t)$. $V'(7)=100\pi S'(7) \approx 602.218$ mm^3/day.

(d) $D(0)=M'(0)-S'(0)=0.825-1.5<0$ and $D(60)=M'(60)-S'(60)=72.825-3.448>0$.[1] Since both $M'(t)$ and $S'(t)$ are continuous functions, $D(t)$ is continuous. By the Intermediate Value Theorem, $D(t)=0$ for some *t* between 0 and 60. At that point in time, $M'(t)=S'(t)$.

Notes:

1. Note that we are given expressions for $S'(t)$ and $M(t)$. It might be easier to enter $M(t)$ into your calculator, then get $M'(0)$ and $M'(60)$ numerically.

Question AB-2 (Form B)

(a) $\lim_{t\to5^+} r(t) = 1000e^{-0.2\cdot5} = \frac{1000}{e}$ and $\lim_{t\to5^-} r(t) = \frac{3000}{8}$. Since $\lim_{t\to5^+} r(t) \neq \lim_{t\to5^-} r(t)$, $\lim_{t\to5} r(t)$ does not exist, so r is not continuous at $t = 5$.

(b) The average value of $r(t)$ is
$\frac{1}{8}\int_0^8 r(t)\,dt = \frac{1}{8}\left(\int_0^5 \frac{600t}{t+3}\,dt + \int_5^8 1000e^{-2t}dt\right)$ 1 🖩 $\approx \frac{1}{8}(1234.5073 + 829.9146) \approx$
258.053 liters per hour.

(c) $r'(3)$ 🖩 $= 50$ liters per hour per hour. At $t = 3$ hours, the drainage rate is increasing at the rate of 50 liters per hour per hour.

(d) $12000 - \int_0^A r(t)\,dt = 9000$.

Notes:

1. Calculating an integral of a piecewise-defined discontinuous function is rare, if not unprecedented, in a free response problem. Nonetheless, integrating such functions is obviously a skill that is tested.

Question AB-3 (Form B)

(a) Area = $\int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 + \frac{1}{2}\cdot 2\cdot 2 = \frac{16}{3} + 2 = \frac{22}{3}$. [1]

(b) Volume = $\int_0^2 2y(6-y-y^2)\,dy$.

(c) The graph of g has the slope -1. So we must have
$f'(x)=1 \;\Rightarrow\; \frac{1}{2\sqrt{x}}=1 \;\Rightarrow\; x=\frac{1}{4},\; y=\frac{1}{2}$.

Notes:

1. It is easier to use geometry — the area of the triangle — for $\int_4^6 g(x)\,dx$. Otherwise,
$\int_4^6 (6-x)\,dx = \left(6x-\frac{x^2}{2}\right)\Big|_4^6 = \left(36-\frac{36}{2}\right)-\left(24-\frac{16}{2}\right)=2$.

Question AB-4 (Form B)

(a) f has a critical point where $f'(x)=(4-x)x^{-3}=0 \;\Rightarrow\; x=4$. This is a relative maximum for f since $f'(x)$ changes sign from positive to negative at $x = 4$.

(b) $f'(x)=4x^{-3}-x^{-2} \;\Rightarrow\; f''(x)=-12x^{-4}+2x^{-3}=x^{-4}(-12+2x)$. $f''(x)<0$ if and only if $0 < x < 6$. Therefore, the graph of f is concave down on $(0, 6)$. [1]

(c) $f(x)=f(1)+\int_1^x f'(t)\,dt = 2+\int_1^x \left(4t^{-3}-t^{-2}\right)dt =$

$2+4\cdot\frac{t^{-2}}{-2}-\frac{t^{-1}}{-1}\Big|_1^x = 2-\frac{2}{x^2}+\frac{1}{x}-(-2+1) = 3-\frac{2}{x^2}+\frac{1}{x}$.

Notes:

1. Or $(0, 6]$. $x = 0$ must be excluded, because it is not in the domain of f.

Question AB-5 (Form B)

(a) $a(5) = v'(5) \approx \dfrac{v(10) - v(0)}{10 - 0} = \dfrac{2.3 - 2}{10} = 0.03$ meters per second per second.

(b) $\int_0^{60} |v(t)|\, dt$ is the total distance in meters Ben traveled on his unicycle during the 60 seconds. $\int_0^{60} |v(t)|\, dt \approx 2.0 \cdot (10 - 0) + 2.3 \cdot (40 - 10) + 2.5 \cdot (60 - 40)$ meters.[1]

(c) Yes. The Mean Value Theorem applied to the time interval [40, 60] guarantees a time t when $v(t) = B'(t) = \dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2$.

(d) $2L(t)L'(t) = 2B(t)B'(t) \quad \Rightarrow \quad L'(t) = \dfrac{B(t)B'(t)}{L(t)}$. At t = 40,

$\left(L(40)\right)^2 = 144 + \left(B(40)\right)^2 \quad \Rightarrow \quad L(40) = \sqrt{144 + 81} = \sqrt{225} = 15$. So

$L'(40) = \dfrac{9 \cdot 2.5}{15}$ meters per second.[2]

Notes:

1. And leave it at that to save time and avoid arithmetic mistakes. (This is equal to 139 meters.)
2. = 1.5 m/sec.

Courtesy John Mahoney, ap-calc EDG moderator, http://www.nthf.org/inductee/05mahoney.htm.

Question AB-6 (Form B)

(a) $\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} g(x)\,dx - \int_{-2\pi}^{4\pi} \cos\left(\frac{x}{2}\right)dx = \frac{1}{2}\cdot 6\pi \cdot 2\pi - 2\sin\left(\frac{x}{2}\right)\Bigg|_{-2\pi}^{4\pi} = 6\pi^2$.

(b) *f* has a critical point at $x = 0$ since $g'(0)$ doesn't exist and so $f'(0)$ doesn't exist. For $x \neq 0$, $f'(x) = g'(x) + \frac{1}{2}\cdot\sin\left(\frac{x}{2}\right)$. On the interval $(-2\pi, 0)$, $f'(x) = 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right)$, so the equation $f'(x) = 0$ has no solutions. On the interval $(0, 4\pi)$, $f'(x) = -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right)$. $f'(x) = 0$ at $x = \pi$, where $1 = \sin\left(\frac{x}{2}\right)$. The answer is: *f* has two critical points on $(-2\pi, 4\pi)$, at $x = 0$ and at $x = \pi$.

(c) $h'(x) = 3g(3x) \Rightarrow h'\left(-\frac{\pi}{3}\right) = 3g(-\pi) = 3\pi$.

2011 BC (Form B)
AP Calculus Free-Response
Solutions and Notes

Question BC-1 (Form B)

See AB Question 1 (Form B).

Question BC-2 (Form B)

(a) Area = $\frac{1}{2}\int_{\pi/2}^{\pi}\left(r\left(\theta\right)\right)^2 d\theta$ 🖩 ≈ 47.513.

(b) Solving $r\left(\theta\right)\cos\left(\theta\right)=-3$, we get 🖩 $\theta \approx 2.017$. At this angle, $y=r\left(\theta\right)\sin\left(\theta\right)$ 🖩 ≈ 6.272.

(c) $y=r\left(\theta\right)\sin\left(\theta\right)=\left(3\theta+\sin\theta\right)\sin\theta \;\Rightarrow\; \frac{dy}{dt}=\frac{d}{d\theta}\left(\left(3\theta+\sin\theta\right)\sin\theta\right)\frac{d\theta}{dt}$. At $\theta=\frac{2\pi}{3}$, $\frac{dy}{dt}$ 🖩 $\approx -1.40954\cdot 2 \approx -2.819$. This means that when $\theta=\frac{2\pi}{3}$, the *y*-coordinate of the particle is decreasing at the rate of 2.819 units of length per unit of time.

Question BC-3 (Form B)

See AB Question 3 (Form B).

Question BC-4 (Form B)

(a) The average value of *f* on [0, 5] is $\frac{1}{5}\int_0^5 f(x)\,dx = \frac{1}{5}(-10) = -2$.

(b) $\int_0^{10} f(x)dx = -10 + 27 = 17 \;\Rightarrow\; \int_0^{10}(3f(x)+2)dx = 3\cdot 17 + 20 = 71$.

(c) Since $g'(x) = f(x)$ and $f(x) < 0$ on (0, 5), *g* is decreasing on [0, 5]. The graph of *g* is concave up where $g' = f$ is increasing, that is when $3 < x < 8$. *g* is both concave up and decreasing on (3, 5).[1]

(d) The arc length is

$$\int_0^{20}\sqrt{1+(h'(x))^2}\,dx = \int_0^{20}\sqrt{1+\left(f'\left(\frac{x}{2}\right)\right)^2}\,dx = 2\int_0^{10}\sqrt{1+(f'(u))^2}\,du,$$ where

$u = \frac{x}{2}$, $du = \frac{1}{2}dx$. Therefore, the arc length of the graph of *h* from 0 to 20 is twice the arc length of the graph of *f* on [0, 10]. The latter is $11 + 18 = 29$, so the arc length of the graph of *h* from 0 to 20 is equal to 58.

Notes:

1. Or [3, 5].

Question BC-5 (Form B)

See AB Question 5 (Form B).

Question BC-6 (Form B)

(a) $\ln(1+x^3)=x^3-\dfrac{x^6}{2}+\dfrac{x^9}{3}-\dfrac{x^{12}}{4}+...+(-1)^{n+1}\cdot\dfrac{x^{3n}}{n}+....$

(b) At x = 1, the series is $1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+...+(-1)^{n+1}\cdot\dfrac{1}{n}+....$ It converges by the alternating series convergence test. At $x=-1$, the series is $-1-\dfrac{1}{2}-\dfrac{1}{3}-\dfrac{1}{4}-...-\dfrac{1}{n}-....$ It diverges, because it is the negative of the harmonic series. The interval of convergence is $(-1,1]$.

(c) The first four terms of the series for $f'(t)$ are

$$f'(t)=3t^2-\frac{6t^5}{2}+\frac{9t^8}{3}-\frac{12t^{11}}{4}+...=3t^2-3t^5+3t^8-3t^{11}+...\ \Rightarrow$$

$f'(t^2)=3t^4-3t^{10}+3t^{16}-3t^{22}+....$ $g(1)\approx\int_0^1\left(3t^4-3t^{10}\right)dt=\left(\dfrac{3t^5}{5}-\dfrac{3t^{11}}{11}\right)\Bigg|_0^1=\dfrac{3}{5}-\dfrac{3}{11}.$

(d) The alternating series error does not exceed the absolute value of first omitted term, $\displaystyle\int_0^1 3t^{16}\,dt=\frac{3}{17}<\frac{1}{5}.$