Be Prepared

for the

AP

Calculus Exam

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Chapter 10. Annotated Solutions to Past Free-Response Questions

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2007 AB
AP Calculus Free-Response
Solutions and Notes

Question 1

Solving $\frac{20}{1+x^2}=2$, the graphs intersect at $x=-3$ and $x=3$.

(a) Area = $\int_{-3}^{3}\frac{20}{1+x^2}-2\;dx \approx 37.962$. [1]

(b) Volume = $\pi\int_{-3}^{3}\left(\frac{20}{1+x^2}\right)^2-2^2\;dx$ [2] ≈ 1871.190.

(c) Volume = $\frac{\pi}{2}\int_{-3}^{3}\left(\frac{\frac{20}{1+x^2}-2}{2}\right)^2\;dx$ [3] ≈ 174.268.

Notes:

1. Use your calculator to evaluate the integrals; don't bother trying to antidifferentiate.

2. Using washers

3. $\frac{20}{1+x^2}-2$ is the diameter of the semicircle.

Question 2

Let $V(t)$ be the amount of water in the tank at t hours.

(a) $\int_0^7 f(t)\,dt \approx 8264$ gallons.

(b) $V'(t) = f(t) - g(t)$ for $0 < t < 3$ and $3 < t < 7$.[1] From the graphs of $f(t)$ and $g(t)$ and the given intersection points, $V'(t) = f(t) - g(t) < 0$ when $0 < t < 1.617$ and $3 < t < 5.076$. Therefore, $V(t)$ is decreasing on $0 \le t \le 1.617$ and $3 \le t \le 5.076$.[2]

(c) $V(t)$ is decreasing on $[0, 1.617]$ and $[3, 5.076]$ (see Part (b)). It is increasing on $[1.617, 3]$ and $[5.076, 7]$, because $V'(t) = f(t) - g(t) > 0$ inside these intervals. Therefore, $V(t)$ can reach its global maximum only at $t = 0$, $t = 3$, or $t = 7$.

$V(0) = 5000$; $V(3) = 5000 + \int_0^3 f(t) - 250\,dt \approx 5126.591$;

$V(7) = V(3) + \int_3^7 f(t) - 2000\,dt \approx 4513.806$. The global maximum is approximately 5127 gallons[3] at $t = 3$.

Notes:

1. Don't worry about the physics of it — you can waste a lot of time. The model is good enough for the situation where someone "pulled a plug" very quickly at $t = 3$, causing a jump in the <u>rate of change</u> of $V(t)$. $V(t)$ itself remains continuous.

2. You might be wondering: How can we say that $V(t)$ is decreasing at $t = 3$ when $V'(t)$ is undefined at $t = 3$? Well, we are not saying that. What we are saying is that $V(t)$ is decreasing <u>on the interval</u> $[3, 5.076]$, which means that for any t_1 and t_2, such that $3 \le t_1 < t_2 \le 5.076$, $V(t_2) < V(t_1)$. It is not clear at this point whether $0 < t < 1.617$ and $3 < t < 5.076$ (open intervals) will get full credit.

3. Be attentive to the units.

Question 3

(a) $h(1) = f\left(g(1)\right) - 6 = f(2) - 6 = 9 - 6 = 3$.
$h(3) = f\left(g(3)\right) - 6 = f(4) - 6 = -1 - 6 = -7$. h is differentiable for all real numbers and, therefore, continuous on $[1, 3]$. $h(1) > -5 > h(3)$, therefore, by the Intermediate Value Theorem, there must exist a number r on $(1, 3)$ such that $h(r) = -5$.

(b) h is differentiable on $[1, 3]$. Therefore, by the Mean Value Theorem there must exist a number c such that $1 < c < 3$ and $h'(c) = \dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{2} = -5$.

(c) $w(x) = \int_1^{g(x)} f(t)\, dt \Rightarrow$ [1] $w'(x) = f\left(g(x)\right) \cdot g'(x) \Rightarrow$
$w'(3) = f(g(3))g'(3) = (-1)2 = -2$.

(d) $g(1) = 2 \Rightarrow g^{-1}(2) = 1$. $\dfrac{d}{dx}\left(g^{-1}(x)\right)\Big|_{x=2} = \dfrac{1}{g'(1)} = \dfrac{1}{5}$.

An equation of the tangent line at $x = 2$ is $y = 1 + \dfrac{1}{5}(x - 2)$. [2]

Notes:

1. Applying the chain rule

2. Or: $y - 1 = \dfrac{1}{5}(x - 2)$

Question 4

(a) $x(t) = e^{-t}\sin t, \quad 0 \le t \le 2\pi$

$x'(t) = e^{-t}\cos t - e^{-t}\sin t = e^{-t}(\cos t - \sin t)$

For $0 < t < 2\pi$, $x'(t)$ changes from negative to positive[1] only at $t = \frac{5\pi}{4}$, so $x(t)$ has a local minimum only at $t = \frac{5\pi}{4}$. Comparing the values $x(0) = 0$, $x\left(\frac{5\pi}{4}\right) = e^{\frac{-5\pi}{4}}\left(-\frac{\sqrt{2}}{2}\right)$, and $x(2\pi) = 0$, we conclude that the particle is farthest to the left[2] at $t = \frac{5\pi}{4}$.

(b) $x''(t) = e^{-t}(-\sin t - \cos t) - e^{-t}(\cos t - \sin t) = -2e^{-t}\cos t$

$A\left(-2e^{-t}\cos t\right) + e^{-t}(\cos t - \sin t) + e^{-t}\sin t = 0 \;\Rightarrow\; e^{-t}\cos t(-2A+1) = 0 \;\Rightarrow$

$-2A + 1 = 0 \;\Rightarrow\; A = \frac{1}{2}$.

Notes:

1. You might want to sketch the graphs of $\cos t$ and $\sin t$ on $[0, 2\pi]$ to see that.

2. That is, $x(t)$ is the smallest.

Question 5

(a) $r(5) = 30$ and $r'(5) = 2$, so the tangent line to the graph of $y = r(t)$ at $t = 5$ is $y = 30 + 2(x - 5)$. When $x = 5.4$, $y = 30 + 2(5.4 - 5) = 30 + 0.8 = 30.8$. So $r(5.4) \approx 30.8$. Since the graph of *r* is concave down, the tangent line is above the graph, so the estimate is greater than the actual value of $r(5.4)$.

(b) $\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{t=5} = 4\pi \cdot 30^2 \cdot 2.0$ [1] ft^3/minute.

(c) Right Riemann sum approximation of $\int_0^{12} r'(t)\,dt$ is

$2 \cdot 4.0 + 3 \cdot 2.0 + 2 \cdot 1.2 + 4 \cdot 0.6 + 1 \cdot 0.5$. [1] $\int_0^{12} r'(t)\,dt = r(12) - r(0)$ gives the net change in the radius of the balloon (in feet) from $t = 0$ minutes to $t = 12$ minutes.

(d) Since the graph of *r* is concave down, $r'(t)$ is decreasing. Therefore, a right Riemann sum approximation is less than the integral.

Notes:

1. No need to simplify the answer.

Question 6

(a) $f'(x)=\dfrac{k}{2\sqrt{x}}-\dfrac{1}{x}$

$f''(x)=-\dfrac{k}{4\sqrt{x^3}}+\dfrac{1}{x^2}$

(b) $f'(1)=0 \Rightarrow \dfrac{k}{2}-1=0 \Rightarrow k=2$

When $k=2$, $f''(1)=-\dfrac{2}{4}+1>0$, so f has a relative minimum at $x=1$.

(c) For x to be an inflection point we need

$f''(x)=0 \Rightarrow -\dfrac{k}{4\sqrt{x^3}}+\dfrac{1}{x^2}=0 \Rightarrow \dfrac{x^2}{\sqrt{x^3}}=\dfrac{4}{k} \Rightarrow x=\left(\dfrac{4}{k}\right)^2$ and $k>0$.[1] For the inflection point to be on the x-axis, we need $f\left(\left(\dfrac{4}{k}\right)^2\right)=k\cdot\dfrac{4}{k}-\ln\left(\left(\dfrac{4}{k}\right)^2\right)=0 \Rightarrow$

$\ln\left(\left(\dfrac{4}{k}\right)^2\right)=4 \Rightarrow \ln\left(\dfrac{4}{k}\right)=2 \Rightarrow \dfrac{4}{k}=e^2 \Rightarrow k=\dfrac{4}{e^2}$.[2]

Notes:

1. Because $x>0$.

2. An alternative solution:

$$\left.\begin{array}{l} k\sqrt{x}-\ln x=0 \Rightarrow k=\dfrac{\ln x}{\sqrt{x}} \\ \dfrac{-k}{4\sqrt{x^3}}+\dfrac{1}{x^2}=0 \Rightarrow k=\dfrac{4}{\sqrt{x}} \end{array}\right\} \Rightarrow \ln x=4 \Rightarrow x=e^4 \Rightarrow k=\dfrac{4}{e^2}$$

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AP Calculus Free-Response
Solutions and Notes

Question 1

See AB Question 1.

Question 2

See AB Question 2.

Question 3

(a) Area of R =

$$\int_0^{2\pi/3}\frac{(2)^2}{2}\,d\theta+\int_{2\pi/3}^{4\pi/3}\frac{(3+2\cos\theta)^2}{2}\,d\theta+\int_{4\pi/3}^{2\pi}\frac{(2)^2}{2}\,d\theta=$$

$$\frac{4\pi}{3}+\int_{2\pi/3}^{4\pi/3}\frac{(3+2\cos\theta)^2}{2}\,d\theta+\frac{4\pi}{3}\approx 10.370.$$ [1]

(b) $\frac{dr}{dt}=\frac{dr}{d\theta}=-2\sin\theta$. When $\theta=\frac{\pi}{3}$, $\frac{dr}{dt}=-2\sin\left(\frac{\pi}{3}\right)=-\sqrt{3}$.
At this time, the particle is getting closer to the origin (r is decreasing).

(c) $y=r\sin\theta=(3+2\cos\theta)\cdot\sin\theta \Rightarrow \frac{dy}{d\theta}=(3+2\cos\theta)\cdot\cos\theta+\sin\theta\cdot(-2\sin\theta)$. [2]

At $\theta=\frac{\pi}{3}$, this is $\left(3+2\cdot\frac{1}{2}\right)\cdot\frac{1}{2}-2\left(\frac{\sqrt{3}}{2}\right)^2=2-\frac{3}{2}=\frac{1}{2}$. The y-coordinate of the particle is increasing, so the particle is moving up.

Notes:

1. Or use the symmetry: Area of R = $2\cdot\left[\int_0^{2\pi/3}\frac{(2)^2}{2}\,d\theta+\int_{2\pi/3}^{\pi}\frac{(3+2\cos\theta)^2}{2}\,d\theta\right]$.

2. You don't have to calculate the symbolic derivative: you can find it at $\theta=\frac{\pi}{3}$ numerically on your calculator.

Question 4

(a) $f'(e) = e^2$. Tangent line at the point $(e, 2)$ is $y - 2 = e^2(x - e)$.

(b) $f'(x) = x^2 \ln x$ is increasing on the interval $1 < x < 3$, because both x^2 and $\ln x$ are positive and increasing on that interval. Therefore, the graph of f is concave up.

(c) $f(x) = \int f'(x)\,dx = \int x^2 \ln x\,dx$ [1]

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$.

$f(e) = 2 \Rightarrow f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 - \frac{1}{3}e^3 + \frac{1}{9}e^3 + 2$.

Notes:

1. By parts

Question 5

See AB Question 5.

Question 6

(a) $1+\left(-x^2\right)+\dfrac{\left(-x^2\right)^2}{2!}+\dfrac{\left(-x^2\right)^3}{3!}+\ldots+\dfrac{\left(-x^2\right)^n}{n!}+\ldots = 1-x^2+\dfrac{x^4}{2}-\dfrac{x^6}{6}+\ldots+(-1)^n\dfrac{x^{2n}}{n!}+\ldots$

(b) $\lim\limits_{x\to 0}\dfrac{1-x^2-f(x)}{x^4}=\lim\limits_{x\to 0}\dfrac{1-x^2-\left(1-x^2+\dfrac{x^4}{2}-\dfrac{x^6}{6}+\ldots\right)}{x^4}=$

$\lim\limits_{x\to 0}\dfrac{-\dfrac{x^4}{2}+\dfrac{x^6}{6}+\ldots}{x^4}=\lim\limits_{x\to 0}\left(-\dfrac{1}{2}+\dfrac{x^2}{6}+\ldots\right)=-\dfrac{1}{2}.$

(c) $\int_0^x e^{-t^2}\,dt=\int_0^x 1-t^2+\dfrac{t^4}{2}-\dfrac{t^6}{6}+\ldots dt = x-\dfrac{x^3}{3}+\dfrac{x^5}{10}-\dfrac{x^7}{42}+\ldots.$ $\int_0^{1/2} e^{-t^2}dt\approx\dfrac{1}{2}-\dfrac{1}{3}\cdot\dfrac{1}{8}.$ [1]

(d) When $x=\dfrac{1}{2}$, the series in Part (c) is alternating with decreasing absolute values of terms, and the n-th term approaches 0 as $n\to\infty$. By the alternating series error bound, the magnitude of the error using the first two terms is less than the third non-zero term, which is $\dfrac{1}{10}\cdot\left(\dfrac{1}{2}\right)^5=\dfrac{1}{320}$, which is less than $\dfrac{1}{200}$.

Notes:

1. Do not simplify.

2007 AB (Form B)
AP Calculus Free-Response
Solutions and Notes

Question 1

The line $y = 2$ intersects the graph of $y = e^{2x-x^2}$ at $x = 0.446057$ and $x = 1.553943$. Let $a = 0.446057$ and $b = 1.553943$.[1] The line $y = 1$ intersects the graph at $x = 0$ and at $x = 2$.

(a) $A_R = \int_a^b \left(e^{2x-x^2} - 2\right) dx \approx 0.514$.[2]

(b) $A_S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx - A_R = \approx 1.546$.[3]

(c) $V = \pi \int_a^b \left(e^{2x-x^2} - 1\right)^2 - \left(2-1\right)^2 \, dx$.

Notes:

1. Store the intersection points in calculator variables and use those variables when calculating the integrals. See *Be Prepared*, page 256.

2. Save the result, before rounding, in your calculator — for Part (b).

3. Alternative solution: since the curve is symmetrical about the line $x = 1$,
   $A_S = 2\int_0^a \left(e^{2x-x^2} - 1\right) dx + \left(b-a\right)\left(2-1\right)$.

Question 2

(a) Acceleration is $a(t) = v'(t)$. So $a(3) = v'(3) \approx -5.467$.[1]

(b) Total distance traveled is $\int_0^3 |v(t)|\,dt \approx 1.702$.

(c) $x(3) = x(0) + \int_0^3 v(u)\,du \approx 5.774$.

(d) The position of the particle at time t is given by $x(t) = x(0) + \int_0^t v(u)\,du$.

The rightmost position can occur at a time when the velocity changes sign from positive to negative, $t = \sqrt{\pi}$ or $t = \sqrt{3\pi}$, or at one of the endpoints of the time interval, $t = 0$ or $t = \sqrt{5\pi}$. Evaluating $x(t)$ at these points gives $x(0) = 5$, $x(\sqrt{\pi}) = 5.895$, $x(\sqrt{3\pi}) = 5.788$, and $x(\sqrt{5\pi}) = 5.752$. The rightmost position of the particle occurs at $t = \sqrt{\pi}$.

Notes:

1. Or, if you use symbolic differentiation, $6\cos(9)$ — then you can leave your answer in this form.

Question 3

(a) $W'(20)$ 🖩 $\approx -0.286\ ^{\circ}\text{F/mph}$. When the air temperature is 32 °F and the wind velocity is 20 mph, the wind chill is decreasing at the rate of 0.286 °F/mph.

(b) Average rate of change of W is $\dfrac{W(60)-W(5)}{60-5}$ 🖩 $\approx -0.254\ ^{\circ}\text{F/mph}$.

Solving $W'(v) = -0.254$ gives $v \approx 23.011$ mph.[1]

(c) $\dfrac{dW}{dt} = \dfrac{dW}{dv}\cdot\dfrac{dv}{dt} = -22.1\cdot 0.16v^{-0.84}\cdot 5$

At $t = 3$, $v = 20 + 3\cdot 5 = 35$ mph, and $\left.\dfrac{dW}{dt}\right|_{t=3} = (-22.1)\cdot 0.16\cdot\left(35^{-0.84}\right)\cdot 5\ ^{\circ}\text{F/hour}$.

Notes:

1. The existence of such value of v is guaranteed by the Mean Value Theorem.

Question 4

(a) $x=-3$ and $x=4$, since f' changes sign from positive to negative at these points.

(b) $x=-4$, $x=-1$, and $x=2$, since f' changes from increasing to decreasing at $x=-4$ and $x=2$ and changes from decreasing to increasing at $x=-1$.

(c) Slope is positive on the graph of *f* when f' is positive. The graph of *f* is concave up when f' is increasing. These two conditions are met for $-5<x<-4$ and $1<x<2$.

(d) $f(x)=3+\int_1^x f'(t)\,dt$

f could have its absolute minimum at the endpoints, $x=-5$ and $x=5$, or at $x=1$, where f' changes sign from negative to positive. From the graph of f',

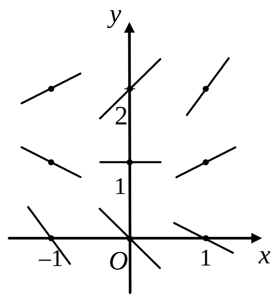
$f(-5)=3+2\pi-\frac{\pi}{2}$ [1] and $f(5)=5.5$. $f(1)=3$. The absolute minimum is 3.

Notes:

1. We integrate from $x = 1$, so an area for $x < 1$ is entered with the opposite sign.

Question 5

(a)



(b) $\dfrac{d^2y}{dx^2} = \dfrac{1}{2} + \dfrac{dy}{dx} = \dfrac{1}{2} + \dfrac{1}{2}x + y - 1 = \dfrac{1}{2}x + y - \dfrac{1}{2}$

If $y > \dfrac{1}{2} - \dfrac{1}{2}x$, then $\dfrac{d^2y}{dx^2} > 0$, and the graph of the solution is concave up.

$y > \dfrac{1}{2} - \dfrac{1}{2}x$ describes half-plane above the line $y = \dfrac{1}{2} - \dfrac{1}{2}x$.

(c) $x = 0$ and $y = 1$, so $\dfrac{dy}{dx} = \dfrac{1}{2} \cdot 0 + 1 - 1 = 0$. At this point, $\dfrac{d^2y}{dx^2} = \dfrac{1}{2} + \dfrac{dy}{dx} = \dfrac{1}{2} > 0$.

By the second derivative test, *f* has a relative minimum at this point.

(d) If $y = mx + b$, then $\dfrac{dy}{dx} = m$. If this is a solution, then

$m = \dfrac{1}{2}x + mx + b - 1 = \left(\dfrac{1}{2} + m\right)x + b - 1$. For these to be equal on some interval, we must have $\dfrac{1}{2} + m = 0$ and $b - 1 = m$. Thus, $m = -\dfrac{1}{2}$ and $b = \dfrac{1}{2}$.

Question 6

(a) Since it is differentiable, *f* must also be continuous. The Mean Value Theorem applied to *f* on [2, 5] guarantees the existence of *c* in (0, 5) such that $f'(c) = \dfrac{f(5) - f(2)}{5-2} = \dfrac{-3}{3} = -1$.

(b) $g'(x) = f'\big(f(x)\big) \cdot f'(x)$, so $g'(2) = f'\big(f(2)\big) \cdot f'(2) = f'(5) \cdot f'(2)$ and $g'(5) = f'\big(f(5)\big) \cdot f'(5) = f'(2) \cdot f'(5)$. Thus $g'(2) = g'(5)$.

Since *g* is the composition of twice-differentiable functions, g' must be continuous on [2, 5] and differentiable on (2, 5). The Mean Value Theorem applied to g' on [2, 5] guarantees the existence of *k* in (2, 5) such that $g''(k) = \dfrac{g'(5) - g'(2)}{5-2} = 0$ because $g'(2) = g'(5)$.

(c) If $f''(x) = 0$ for all *x*, then f' must be a constant and *f* must be a linear function. Let $f(x) = mx + b$. Then $g(x) = m \cdot (mx + b) + b$ is also linear, and $g''(x) = 0$ for all *x*.[1] Thus, g'' never changes sign, and the graph of *g* has no points of inflection.

(d) $h(5) = f(5) - 5 = -3$ and $h(2) = f(2) - 2 = 3$. Since *h* is the difference of two continuous functions, it is continuous. Since –3 < 0 < 3, by the Intermediate Value Theorem, there is a number *r* in (2, 5) such that $h(r) = 0$.

Notes:

1. Alternative solution:
$g'(x) = f'\big(f(x)\big) \cdot f'(x) \Rightarrow g''(x) = f''\big(f(x)\big) \cdot \left[f'(x)\right]^2 + f'\big(f(x)\big) \cdot f''(x)$. If $f''(x) = 0$ for all *x*, then $g''(x) = 0$ for all *x*.

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Solutions and Notes

Question 1

See AB Question 1.

Question 2

(a) Speed at $t = 4$ is $\left.\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\right|_{t=4} = \sqrt{\left(\arctan\left(\frac{4}{5}\right)\right)^2 + \left(\ln(17)\right)^2}$.[1]

(b) Distance traveled = $\int_0^4 \sqrt{\left(\arctan\left(\frac{t}{1+t}\right)\right)^2 + \left(\ln\left(t^2+1\right)\right)^2}\, dt \approx 6.423$.

(c) $x(4) = x(0) + \int_0^4 \frac{dx}{dt}\, dt = -3 + \int_0^4 \arctan\left(\frac{t}{1+t}\right) dt \approx -0.892$.

(d) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\ln\left(t^2+1\right)}{\arctan\left(\frac{t}{1+t}\right)} = 2$ when $t \approx 1.357663$ [2] ≈ 1.358 . At this time, the acceleration vector is $\vec{a} = \left.\left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\right)\right|_{t=1.357663} \approx (0.135,\ 0.955)$.

Notes:

1. You can leave it at that.

2. Store this result in your calculator.

Question 3

See AB Question 3.

Question 4

See AB Question 4.

Question 5

(a) $\dfrac{d^2y}{dx^2} = 3 + 2\dfrac{dy}{dx} = 3 + 2(3x + 2y + 1) = 6x + 4y + 5$

(b) $\dfrac{dy}{dx} = m + re^{rx} = 3x + 2(mx + b + e^{rx}) + 1 = 2b + 1 + (3 + 2m)x + 2e^{rx}$.
Equating the coefficients of the like terms we get $2 = r$, $3 + 2m = 0$, and
$2b + 1 = m \Rightarrow r = 2,\ m = -\dfrac{3}{2},\ b = -\dfrac{5}{4}$. [1]

(c) $x_0 = 0$ and $y_0 = -2$. $f\left(\dfrac{1}{2}\right) \approx -2 + (3\cdot 0 + 2\cdot(-2) + 1)\cdot\dfrac{1}{2} = -\dfrac{7}{2}$. Using $\left(\dfrac{1}{2}, -\dfrac{7}{2}\right)$,
$f(1) \approx -\dfrac{7}{2} + \left(3\cdot\dfrac{1}{2} + 2\cdot\left(-\dfrac{7}{2}\right) + 1\right)\cdot\dfrac{1}{2}$. [2]

(d) Starting with $x_0 = 0$, $y_0 = k$ and using $\Delta x = 1$, we get
$f(1) \approx k + (3\cdot 0 + 2\cdot k + 1)\cdot 1 = 3k + 1$. For this to be 0, $k = -\dfrac{1}{3}$.

Notes:

1. This assumes $r \neq 0$. Another possible solution is $r = 0,\ m = -\dfrac{3}{2},\ b = -\dfrac{9}{4}$.

2. $f(1) \approx -\dfrac{23}{4}$, but you don't have to simplify.

Question 6

(a) By substitution into the series for e^x, the Taylor Series is

$$6\cdot\left(1+\frac{-x}{3}+\frac{\left(\frac{-x}{3}\right)^2}{2!}+\frac{\left(\frac{-x}{3}\right)^3}{3!}+\ldots+\frac{\left(\frac{-x}{3}\right)^n}{n!}+\ldots\right)=$$

$$6-2x+\frac{6x^2}{9\cdot 2!}-\frac{6x^3}{27\cdot 3!}+\ldots+\frac{6(-x)^n}{3^n\cdot n!}+\ldots$$

(b) Integrating termwise gives $C+6x-x^2+\frac{6x^3}{27\cdot 2!}-\frac{6x^4}{108\cdot 3!}+\ldots+\frac{6(-1)^n x^{n+1}}{3^n\cdot(n+1)!}+\ldots$.

$g(0)=0 \Rightarrow C=0 \Rightarrow g(x)=6x-x^2+\frac{6x^3}{27\cdot 2!}-\frac{6x^4}{108\cdot 3!}+\ldots+\frac{6(-1)^n x^{n+1}}{3^n\cdot(n+1)!}+\ldots$.

(c) $h(x)=e^x$ and $f'(ax)=6\left(-\frac{1}{3}\right)e^{-ax/3}=-2e^{-ax/3}$, so $e^x=-2ke^{-ax/3} \Rightarrow$

$a=-3$ and $k=-\frac{1}{2}$.