Be Prepared
for the
AP
Calculus Exam

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Chapter 10.   Annotated Solutions to Past Free-Response Questions

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2003 AB
AP Calculus Free-Response
Solutions and Notes

Question 1

(a) $y = \sqrt{x}$ and $y = e^{-3x}$ intersect at a point (a, y) where $a \approx 0.238734$.[1]

Area of $R = \int_a^1 \left(\sqrt{x} - e^{-3x}\right) dx \approx 0.443$.

(b) $R = 1 - e^{-3x}$; $r = 1 - \sqrt{x}$;

Volume = $\int_a^1 \pi\left(R^2 - r^2\right) dx = \int_a^1 \pi\left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right) dx \approx 1.424$.

(c) $b = \sqrt{x} - e^{-3x}$; $h = 5\left(\sqrt{x} - e^{-3x}\right)$.

Volume = $\int_a^1 bh\, dx = \int_a^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx \approx 1.554$.

Notes:

1. Store *a* in a named variable in your calculator (see *Appendix: Calculator Skills*).

Question 2

(a) $a(2) = v'(2) \approx 1.587$. Since $v(2) = -3\sin(2) < 0$ and $a(2) > 0$, the velocity and acceleration have different signs[1], so the speed is decreasing.

(b) We must have $v(t) = 0$, so $t = -1$ or $\frac{t^2}{2} = k\pi \Rightarrow t = \pm\sqrt{2k\pi}$. Of these values only $t = \sqrt{2\pi}$ falls into the interval $0 < t < 3$. The velocity indeed changes its sign at that point in time because the sign of $(t + 1)$ does not change and the sign of $\sin\frac{t^2}{2}$ changes when $t = \sqrt{2\pi}$.[1]

(c) Distance traveled = $\int_0^3 |v(t)|\, dt \approx 4.333$.[2]

(d) By the Candidate Test, the maximum distance over $0 \le t \le 3$ can be reached at $t = \sqrt{2\pi}$ or $t = 0$ or $t = 3$. The distance from the origin[3] at $t = \sqrt{2\pi}$ is $\left| x(0) + \int_0^{\sqrt{2\pi}} v(t)\, dt \right| \approx |1 - 3.265| = 2.265$. The distance from the origin at $t = 0$ is 1 and at $t = 3$ is $\left| x(0) + \int_0^3 v(t)\, dt \right| \approx |1 - 2.196| = 1.196$.[4] The maximum is 2.265.

Notes:

1. Justification is required for full credit.
2. This is shorter than $\left| \int_0^{\sqrt{2\pi}} v(t)\, dt \right| + \left| \int_{\sqrt{2\pi}}^3 v(t)\, dt \right|$.
3. From the origin, not from $x(0)$!
4. Another line of reasoning may be used. Since the total distance traveled over $0 \le t \le 3$ is 4.333 and the particle got to the position -2.265, it will be still to the left of the origin at $t = 3$ and therefore the maximum distance from the origin cannot be reached at $t = 3$.

Question 3

(a) $R'(45) \approx \dfrac{R(50)-R(40)}{10} = \dfrac{55-40}{10} = 1.5\,\text{gal/min}^2$.

(b) $R''(45)=0$ since $R'(t)$ has a maximum at $t=45$.[1]

(c) $\int_0^{90} R(t)\,dt \approx 30\cdot 20+10\cdot 30+10\cdot 40+20\cdot 55+20\cdot 65=3700\,\text{gal}$.[2] This approximation is less than the actual value because, for an increasing function, a left-hand Riemann sum underestimates the integral.

(d) $\int_0^{b} R(t)\,dt$ (gal[3]) is the total fuel consumed over the time interval $0 \le t \le b$.

$\dfrac{1}{b}\int_0^{b} R(t)\,dt$ (gal/min[3]) is the average rate of fuel consumption over the time interval $0 \le t \le b$.

Notes:

1. You have to give a correct reason for full credit.
2. An additional check: lengths of the subintervals, 30, 10, 10, 20, 20, must add up to 90.
3. Units must be specified for full credit.

Question 4

(a) f is increasing on [–3, –2] since $f'(x)$ is positive for $-3 < x < -2$ (and nowhere else).

(b) f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$. Therefore, there are two points of inflection: at $x = 0$ and at $x = 2$.

(c) $y = f(0) + f'(0) \cdot x = 3 - 2x$.

(d) $f(-3) = f(0) + \int_0^{-3} f'(x)\,dx = 3 - \int_{-3}^{0} f'(x)\,dx = 3 - \left(\frac{1}{2} - 2\right)$. [1]

$f(4) = f(0) + \int_0^{4} f'(x)\,dx = 3 - \left(2 \cdot 4 - \frac{1}{2}\pi \cdot 2^2\right)$ [1, 2]

Notes:

1. To save time and avoid mistakes, do not simplify further.
2. $2 \cdot 4 - \frac{1}{2}\pi \cdot 2^2$ represents the area of the rectangle minus the area of the semicircle. Since the region is below the x-axis, its contribution to the integral is negative.

Question 5

(a) $\frac{dV}{dt} = \frac{d\left(\pi r^2 h\right)}{dt} = 25\pi \frac{dh}{dt} \Rightarrow -5\pi\sqrt{h} = 25\pi \frac{dh}{dt}$. Therefore, $\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = -\frac{\sqrt{h}}{5}$.

(b) $\frac{dh}{dt} = -\frac{\sqrt{h}}{5} \Rightarrow \frac{dh}{\sqrt{h}} = -\frac{dt}{5} \Rightarrow \int \frac{dh}{\sqrt{h}} = -\int \frac{dt}{5} \Rightarrow 2\sqrt{h} = -\frac{1}{5}t + C$.

$h(0) = 17 \Rightarrow C = 2\sqrt{17}$, $h = \left(\frac{-\frac{1}{5}t + 2\sqrt{17}}{2}\right)^2 = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$.

(c) $h(t) = 0 \Rightarrow -\frac{1}{10}t + \sqrt{17} = 0 \Rightarrow t = 10\sqrt{17}$.

Question 6

(a) Yes. $\lim_{x\to 3^-} f(x) = \sqrt{x+1}\Big|_{x=3} = 2$; $\lim_{x\to 3^+} f(x) = (5-x)\Big|_{x=3} = 2$. Therefore, $\lim_{x\to 3} f(x) = 2 = f(3)$.

(b) $\int_0^5 f(x)dx = \int_0^3 \sqrt{x+1}\,dx + \int_3^5 (5-x)\,dx =$

$$\frac{2}{3}(x+1)^{\frac{3}{2}}\Big|_0^3 + \left(5x - \frac{x^2}{2}\right)\Big|_3^5 = \frac{2}{3}(8-1) + \left(25 - \frac{25}{2} - 15 + \frac{9}{2}\right) = \frac{20}{3}.$$

The average is $\frac{1}{5}\int_0^5 f(x)dx = \frac{4}{3}$.

(c) $g(x)$ is continuous at $x = 3$, so $k\sqrt{3+1} = m\cdot 3 + 2 \;\Rightarrow\; 2k = 3m + 2$.

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$g'(x)$ exists at $x = 3$, so $\frac{k}{4} = m \;\Rightarrow\; k = 4m$. Substituting this expression for k in the first equation, we get $8m = 3m + 2 \;\Rightarrow\; m = \frac{2}{5}$. $k = 4m = \frac{8}{5}$.

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Solutions and Notes

Question 1

See AB Question 1.

Question 2

(a) Both x and y are decreasing along the segment BD of the curve, so both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are negative or zero at point C.

(b) At point B, $\frac{dx}{dt}=0 \Rightarrow \cos\left(\frac{\pi t}{6}\right)=0$ or $\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)=0$. For t > 0,

$\frac{\pi t}{6}=\frac{\pi}{2}+k\pi\ (k=0,1,2,...)$ or $\frac{\pi\sqrt{t+1}}{2}=k\pi\ (k=1,2,...)$. The only possible solution for $0<t<9$ is $t = 3$. 1

(c) $x'(8)=-9\cos\frac{8\pi}{6}\sin\frac{\pi\sqrt{8+1}}{2}=-\frac{9}{2}$. $\frac{y'(8)}{x'(8)}=\frac{5}{9}\Rightarrow y'(8)=-\frac{5}{2}$. The velocity is $\langle x'(8),y'(8)\rangle=\left\langle -\frac{9}{2},-\frac{5}{2}\right\rangle$. The speed is $\sqrt{x'(8)^2+y'(8)^2}=\sqrt{\left(\frac{9}{2}\right)^2+\left(\frac{5}{2}\right)^2}$. 2

(d) Distance = $\left|\int_0^9 x'(t)\,dt\right|$ 🖩 ≈ 39.255.

Notes:

1. From the first equality, $t=3+6k$. $0<t<9$ only when $k = 0$, $t = 3$. From the second equality, $t=4k^2-1$. $0<t<9$ only when $k = 1$, $t = 3$. It is a "coincidence" that $t = 3$ satisfies both equations.
2. Leave it at that to save time and avoid arithmetic mistakes.

Question 3

(a) $\frac{5}{3}y = \sqrt{1+y^2} \Rightarrow \left(\frac{25}{9}-1\right)y^2 = 1 \Rightarrow y = \frac{3}{4}$. $x = \frac{5}{3}y \Rightarrow x = \frac{5}{4}$.

$$\frac{dx}{dy} = \frac{2y}{2\sqrt{1+y^2}} = \frac{\frac{3}{4}}{\sqrt{1+\frac{9}{16}}} = \frac{3}{5}.$$ 🗒1

(b) Area of $S = \int_0^{\frac{3}{4}} \left(\sqrt{1+y^2} - \frac{5}{3}y\right) dy$ 🖩 ≈ 0.346.

(c) $x = r\cos\theta; y = r\sin\theta \Rightarrow$

$$x^2 - y^2 = 1 \Leftrightarrow r^2\cos^2\theta - r^2\sin^2\theta = 1 \Leftrightarrow r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}.$$

(d) Area of $S = \int_0^{\tan^{-1}(3/5)} \frac{r^2}{2} d\theta = \int_0^{\tan^{-1}(3/5)} \frac{1}{2\left(\cos^2\theta - \sin^2\theta\right)} d\theta$. 🗒2

🗒 **Notes:**

1. You might notice that $\frac{2y}{2\sqrt{1+y^2}} = \frac{y}{x} = \frac{3}{5}$.

2. Or $\int_0^{\cot^{-1}(5/3)}$... (same as above).

Question 4

See AB Question 4.

Question 5

See AB Question 5.

Question 6

(a) Since the coefficient of the x term in this Maclaurin series is 0, $f'(0)=0$. Since the coefficient of the x^2 term in this Maclaurin series is $-\frac{1}{3!}$,
$\frac{f''(0)}{2}=-\frac{1}{3!}\Rightarrow f''(0)=-\frac{1}{3}$. The first derivative of f is 0 and the second derivative is negative, so f has a local maximum at $x=0$.

(b) This is a converging alternating series with decreasing terms, so the absolute value of the error is less than the first omitted term: $\left|f(1)-\left(1-\frac{1}{3!}\right)\right|<\frac{1}{5!}<\frac{1}{100}$.

(c) $y'=-2\frac{x}{3!}+4\frac{x^3}{5!}-6\frac{x^5}{7!}+...+(-1)^n 2n\frac{x^{2n-1}}{(2n+1)!}+...$ 1

$$xy'+y=\left(-2\frac{x^2}{3!}+4\frac{x^4}{5!}-6\frac{x^6}{7!}+...+(-1)^n 2n\frac{x^{2n}}{(2n+1)!}+...\right)$$
$$+\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+...+(-1)^n\frac{x^{2n}}{(2n+1)!}+...\right)=$$
$$1-3\frac{x^2}{3!}+5\frac{x^4}{5!}-7\frac{x^6}{7!}+...+(-1)^n(2n+1)\frac{x^{2n}}{(2n+1)!}+...=$$
$$=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+...+(-1)^n\frac{x^{2n}}{(2n)!}+...=\cos x.$$

Notes:

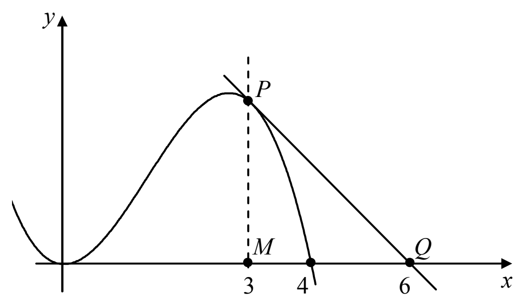
1. This problem does not ask you to write formulas for the general term of each series involved, and it may be easier, in fact, to work without general terms. However, when this question was graded, there was a considerable debate as to how many terms had to be shown for full credit. For those who worked with general terms, there was no such question.
2. If you notice that the given series multiplied by x is the series for $\sin x$ and that $xy'+y=(xy)'$, your proof can be greatly simplified:
$$xy=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...=\sin x\Rightarrow (xy)'=\cos x.$$

2003 AB (Form B)
AP Calculus Free-Response
Solutions and Notes

Question 1

(a) $f'(3) = 8x - 3x^2\Big|_{x=3} = -3$. The tangent line to the graph of $f(x)$ at $x = 3$ is $y = f(3) + f'(3)\cdot(x-3) = 9 + (-3)\cdot(x-3) = -3x + 18$, the same as *l*.

(b) $y = f(x)$ intersects the x-axis at $x = 4$ and *l* intersects the x-axis at $x = 6$. Area of $\triangle MPQ = \frac{1}{2}(6-3)\cdot f(3) = \frac{27}{2}$. Area of $S =$ area of $\triangle MPQ - \int_3^4 f(x)\,dx = \frac{27}{2} - \int_3^4 \left(4x^2 - x^3\right)dx$ 🖩 ≈ 7.917.



(c) Volume = $\pi\int_0^4 \left(4x^2 - x^3\right)^2 dx$ 🖩 ≈ 490.208.

Question 2

(a) Amount of oil pumped in = $\int_0^{12} H(t)\,dt \approx 70.571$ gallons.

(b) Falling. The rate of change of the volume is $\left[H(t)-R(t)\right]\Big|_{t=6} \approx -2.924 < 0$.

(c) Volume = $125+\int_0^{12}\left[H(t)-R(t)\right]dt \approx 122.026$ gallons.

(d) Applying the Candidate Test. $V(0) = 125$. $V(125) = 122.026$ (from Part (c)). Local minima may occur when $\frac{dV}{dt} = 0 \Rightarrow H(t) = R(t) \Rightarrow$ $t \approx 4.790$ or $t \approx 11.318$.

$V(4.790) = 125+\int_0^{4.790}\left[H(t)-R(t)\right]dt \approx 149.4$.[1]

$V(11.318) = 125+\int_0^{11.318}\left[H(t)-R(t)\right]dt \approx 120.7$. The absolute minimum for $V(t)$ is at $t = 11.318$ hours.[2]

Notes:

1. Instead of calculating $V(4.790)$ you could state that $H(t)-R(t)$ is positive for $t < 4.790$, so $V(t)$ can't have a local minimum at $t = 4.790$.

2. Simply graphing $V(t) = \int_0^{t}\left[H(\tau)-R(\tau)\right]d\tau$ and finding its minimum will not get you full credit because reading a maximum or minimum from a graph is not a permitted calculator operation.

Question 3

(a) Average radius = $\frac{1}{360}\int_0^{360}\frac{B(x)}{2}\,dx$.[1]

(b) $\frac{1}{360}\int_0^{360}\frac{B(x)}{2}\,dx \approx \frac{1}{360}\cdot 120\cdot\frac{1}{2}\left[B(60)+B(180)+B(300)\right]=\frac{1}{6}(30+30+24)$.

(c) This integral represents the total volume (in mm^3)[2] of the segment of the blood vessel between x = 125 and x = 275.

(d) By the *MVT*, $B'(x) > 0$ for some $0 \le x \le 60$ and $B'(x) < 0$ for some $60 \le x \le 120$. Therefore (by the *IVT*), $B'(x_1) = 0$ for some $0 \le x_1 \le 120$. Similarly, $B'(x) < 0$ for some $180 \le x \le 300$ and $B'(x) > 0$ for some $300 \le x \le 360$, so $B'(x_2) = 0$ for some $180 \le x_2 \le 360$. Applying the *MVT* (or, its special case, Rolle's theorem) to $B'(x)$ we conclude that $B''(x) = 0$ for some $x_1 < x < x_2$.[3]

Notes:

1. Average radius, not diameter!
2. You must state the units to get full credit.
3. You may notice that $B(60) = B(180)$, so by the MVT (or Rolle's) $B'(x_1) = 0$ for some x_1 from [60, 180]. The same for some x_2 from [240, 360]. From the MVT (or Rolle's) applied to $B'(x)$, we conclude that $B''(x)$ must be equal to 0 somewhere between x_1 and x_2.

Question 4

(a) $a(t) = v'(t) = -e^{1-t}$. $a(3) = v'(3) = -e^{-2}$.

(b) Since $v(3) = -1 + e^{-2} < 0$ and $a(3) < 0$, the velocity and acceleration have the same sign[1], so the speed is increasing.

(c) We must have $v(t) = 0$, so $t = 1$. The velocity indeed changes sign at $t = 1$: $v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$.[2]

(d) Distance traveled = $\int_0^3 |v(t)|\, dt = \int_0^1 \left|-1 + e^{1-t}\right| dt + \int_1^3 \left|-1 + e^{1-t}\right| dt =$

$\left(-t - e^{1-t}\right)\Big|_0^1 - \left(-t - e^{1-t}\right)\Big|_1^3 = \left(-2 + e\right) - \left(-1 - e^{-2}\right) = -1 + e + e^{-2}$.[3]

Notes:

1. Reason is required for full credit.
2. Justification is required for full credit.
3. Or: $x(t) = -t - e^{1-t} \Rightarrow x(0) = -e;\ x(1) = -2;\ x(3) = -3 - e^{-2}$.
   Distance traveled = $\left(x(1) - x(0)\right) + \left(x(1) - x(3)\right) = -1 + e + e^{-2}$.

Question 5

(a) $g(3)=\int_2^3 f(t)\,dt=3$. $g'(3)=f(3)=2$. $g''(3)=f'(3)=\dfrac{2-4}{3-2}=-2$.

(b) The rate of change of g is f. The average of $f(x)$ on $0 \le x \le 3$ is
$\dfrac{1}{3}\int_0^3 f(t)\,dt=\dfrac{1}{3}\left(\int_0^2 f(t)\,dt+\int_2^3 f(t)\,dt\right)=\dfrac{1}{3}(4+3)=\dfrac{7}{3}$.

(c) Again, $g'(c)=f(c)$. The line $y=\dfrac{7}{3}$ intersects the graph of f in two points with x-coordinates in the interval (0, 3).

(d) g' (which is f) changes from increasing to decreasing at $x = 2$ and from decreasing to increasing at $x = 5$. Therefore, there are two inflection points: at $x = 2$ and at $x = 5$.

Question 6

(a) $f''(x) = \frac{d}{dx} f'(x) = \frac{d}{dx} x\sqrt{f(x)} = \sqrt{f(x)} + x\frac{f'(x)}{2\sqrt{f(x)}} = \sqrt{f(x)} + \frac{x^2}{2}$.

$f''(3) = \sqrt{25} + \frac{9}{2}$.

(b) $\frac{dy}{dx} = x\sqrt{y} \Rightarrow \int \frac{dy}{\sqrt{y}} = \int x\,dx \Rightarrow 2\sqrt{y} = \frac{x^2}{2} + C$.

$2\sqrt{25} = \frac{3^2}{2} + C \Rightarrow C = 10 - \frac{9}{2} = \frac{11}{2}$. $y = \left[\frac{1}{2}\left(\frac{x^2}{2} + \frac{11}{2}\right)\right]^2$. [1]

Notes:

1. You have to solve for *y* but stop here, do not simplify any further.

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Solutions and Notes

Question 1

See AB Question 1.

Question 2

(a) Area of $R = \int_0^1 \sqrt{1-(x-1)^2}\,dx + \int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$.

(b) Area of $R = \int_0^1 \left[\sqrt{2-y^2} - \left(1-\sqrt{1-y^2}\right)\right] dy$. 1

(c) Area of $R = \int_0^{\frac{\pi}{4}} \frac{\left(\sqrt{2}\right)^2}{2}\,d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(2\cos\theta)^2}{2}\,d\theta$. 2

Notes:

1. In $x = 1 \pm \sqrt{1-y^2}$ we choose $1-\sqrt{1-y^2}$ because we need $x < 1$.
2. Just set up, do not simplify anything.

Question 3

See AB Question 3.

Question 4

(a) Velocity vector = $\left\langle 6e^{3t} - 7e^{-7t}, 9e^{3t} + 2e^{-2t} \right\rangle$.

The speed = $\sqrt{\left[x'(0)\right]^2 + \left[y'(0)\right]^2} = \sqrt{(-1)^2 + 11^2}$.

(b) $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}$. $\lim\limits_{t \to \infty} \dfrac{dy}{dx} = \dfrac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} = \dfrac{9}{6}$.

(c) The tangent line is horizontal when $\dfrac{dy}{dt} = 0 \Leftrightarrow 9e^{3t} + 2e^{-2t} = 0$. Since $9e^{3t} + 2e^{-2t} > 0$ for all *t*, no such *t* exists.

(d) The tangent line is vertical when

$\dfrac{dx}{dt} = 0 \Leftrightarrow 6e^{3t} - 7e^{-7t} = 0 \Leftrightarrow e^{3t+7t} = \dfrac{7}{6} \Leftrightarrow t = \dfrac{1}{10} \ln\left(\dfrac{7}{6}\right)$.

Question 5

See AB Question 3.

Question 6

(a) $f(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3 + \ldots + \frac{f^{(n)}(2)}{n!}(x-2)^n + \ldots =$

$1 + \frac{2!}{3}(x-2) + \frac{3!}{2!3^2}(x-2)^2 + \frac{4!}{3!3^3}(x-2)^3 + \ldots + \frac{(n+1)!}{n!3^n}(x-2)^n + \ldots =$

$1 + \frac{2}{3}(x-2) + \frac{3}{3^2}(x-2)^2 + \frac{4}{3^3}(x-2)^3 + \ldots + \frac{(n+1)}{3^n}(x-2)^n + \ldots$

(b) By the Ratio Test, the series converges when

$\lim_{n\to\infty}\left|\frac{\frac{n+2}{3^{n+1}}(x-2)^{n+1}}{\frac{n+1}{3^n}(x-2)^n}\right| = \left|\frac{x-2}{3}\right|\lim_{n\to\infty}\frac{n+2}{n+1} = \left|\frac{x-2}{3}\right| < 1$. The series diverges when $\left|\frac{x-2}{3}\right| > 1$. The radius of convergence is 3.

(c) $g(x) = 3 + (x-2) + \frac{1}{3}(x-2)^2 + \frac{1}{3^2}(x-2)^3 + \frac{1}{3^3}(x-2)^4 + \ldots + \frac{1}{3^n}(x-2)^{n+1} + \ldots$

(d) The series for g diverges at $x = -2$ because the n-th term does not approach 0:

$\lim_{n\to\infty}\frac{(-4)^{n+1}}{3^n} = -4\cdot\lim_{n\to\infty}\left(-\frac{4}{3}\right)^n$ does not exist because $\left|-\frac{4}{3}\right| > 1$.