Continental Mathematics League

2018 - 2019

Computer Science Contest

Grades 3-5

The contest consists of three “meets.” Each meet has six questions for 30 minutes.

Note: All parts of a problem must be answered correctly for a student to receive credit for the problem. There is no partial credit.

Computer Science Grades 3-5

Meet 1

1. Fill in the blank.

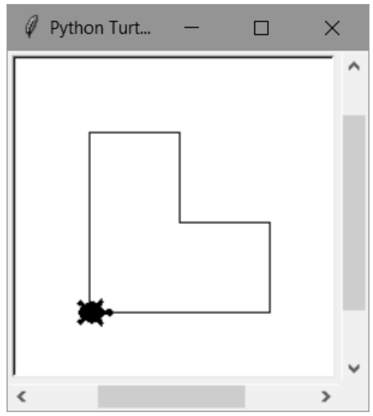
```
>>> 3**2
9
>>> 3**3
27
>>> 3**4
```

____

2. In the sequence of *triangular* numbers, $T_1 = 1$, $T_2 = T_1 + 2$, $T_3 = T_2 + 3$, $\ldots$ That is, $T_n = T_{n-1} + n$ for any $n > 1$. So the sequence goes like this: 1, 3, 6, 10, 15, ... What is the value of T_{10}?

Answer: _______________

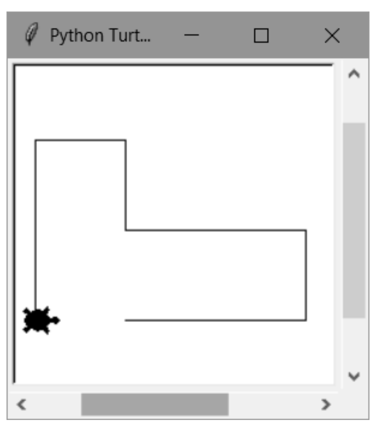
3. Dana wrote a program using Python's Turtle Graphics *module* to draw an "L" like this:



Here is a piece of Dana's program with 14 *statements* that do the actual drawing:

```
# The turtle is at the lower left corner of "L", facing right
pendown()                # Line 1
forward(120); left(90)   # Line 2 forward 120 units, left 90 degrees
forward(60); left(90)    # Line 3
forward(120); right(90)  # Line 4
forward(60); left(90)    # Line 5
forward(60); left(90)    # Line 6
forward(120); left(90)   # Line 7
penup()
```

Unfortunately, Dana's program has a *bug* (error): one of the Python statements is wrong. So Dana got this:



Which line has the bug?

Answer: Line __________

4. In *Pascal's Triangle*, the numbers on the edges are 1's, and any number inside is the sum of the two numbers above it in the previous row, as shown below.

```
 0:              1
 1:            1   1
 2:          1   2   1
                 \ /
 3:        1   3   3   1
...   .................
```

If we count the rows starting from 0 at the top, the sum of the numbers in Row 1 is 2, the sum of the numbers in Row 2 is 4. What is the sum of the numbers in Row 10? Hint: look for a pattern...

Answer: __________

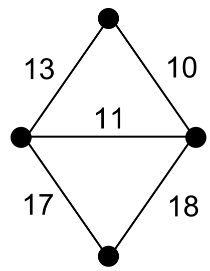
5. Assuming that `yourTurn` is `true`, `gameOver` is `false`, `myScore` is 5 and `yourScore` is 3, what does the following Java code segment print?

```
if (gameOver)
{
  if (yourTurn)
    System.out.println(yourScore + "-" + myScore +
                                  ". Your turn...");
  else
    System.out.println(yourScore + "-" + myScore +
                                  ". Let me think...");
}
else
{
  if (myScore > yourScore)
    System.out.println("Yay, I won!");
  else if (myScore < yourScore)
    System.out.println("Congrats, you win!");
  else
    System.out.println("Tie game");
}
```

(A) `3-5. Your turn...`
(B) `3-5. Let me think...`
(C) `Yay, I won!`
(D) `Congrats, you win!`
(E) `Tie game`

Answer: __________

6.



The *weighted graph* above has four *vertices* connected by *edges*. A number (weight) is assigned to each edge. The total sum of all five weights is 69. We want to remove several edges from the graph in such a way that it remains connected (there is a path from any vertex to any other vertex) and the sum of the remaining weights is the smallest. What is that sum?

Answer: ______________

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Meet 2

1. The programming language Python includes an *interactive* command processor. You type a command at the >>> *prompt*, and the interpreter executes the command. For example:

```
>>> x = 1
>>> print(x)
1
>>> x += 1
>>> print(x)
2
>>> x *= 3
>>> print(x)
6
```

What number is displayed after the following commands are executed?

```
>>> x = 5
>>> x *= 3
>>> x -= 3
>>> x *= 2
>>> print(x)
```

Answer: __________

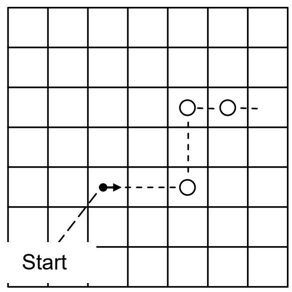
2. In C and many other programming languages, `a % b` gives the remainder when `a` is divided by `b`. For example `17 % 5` is 2, because 17 = 3*5 + 2. If the value of `a` is 13 and the value of `b` is 5, what is the value of `a % (a % b)`?

Answer: __________

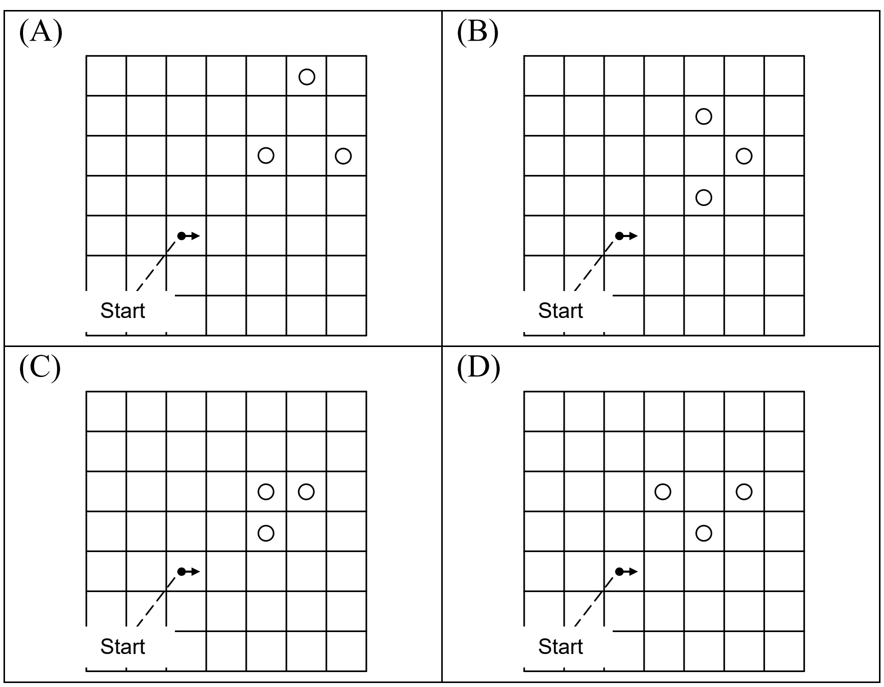
3. Bert is a robot that can move on a floor of square tiles. Bert understands only four commands:

M — Move forward one tile in the current direction
L — Turn left 90 degrees but stay on the same tile
R — Turn right 90 degrees but stay on the same tile
C — Place a "cone" on the current tile

Bert is initially pointing east (to the right). For example, the program M M C L M M R C M C M will produce this configuration of cones:



Which of the following cone configurations will result from the program M M L M C R M L M C M L M L L C M?



Answer: __________

4. Let $s(n)$ be the sum of the digits of n. For example, $s(25) = 7$. What is the smallest positive number n such that $s(n) \geq 10$ while $s(s(n)) < 10$?

Answer: __________

5. Numbers are represented in computers using the *binary* (base-2) number system. The binary system has only two digits, 0 and 1. However, binary representation of a number can be quite long, hard for humans to read. For example, 7531 is written in binary as `1110101101011`. Programmers often use the *hexadecimal* (*hex* for short) number system. It is a base-16 system with 16 digits: 0 - 9, A, B, C, D, E, F. The reason for using the hexadecimal system is that it is easy to convert from binary to hex and from hex to binary, and numbers written in hex have fewer digits and are easier to read. Each hex digit corresponds to a *quad* (group of four) binary digits:

0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

To convert from binary representation of a number to hex, all you have to do is split the digits into quads, starting from the right (adding zeros to the leftmost quad, if necessary), then replace each quad with its hex digit equivalent. For example, `101010` binary ==> `0010 1010` ==> `2A` hex. As we said, 7531 is `1110101101011` in binary. How is 7531 written in hex?

Answer: __________

6. 5 – 3 = 2. This notation is called *Infix Notation*, because the *arithmetic operator* "–" (minus) is written between its *operands*, 5 and 3. In *Reverse Polish Notation* (*RPN*), also known as *Postfix* notation, arithmetic expressions are written with no parentheses: an operator simply follows its operands. For example,

```
4 5 3 - *
```

corresponds to 4 * (5 – 3) and the result is 8. What is the result of the RPN expression below?

```
4 1 3 + 5 2 - * *
```

Answer: __________

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Meet 3

1. `"AXIOM".charAt(3)` returns `'O'`; `"AXIOM".charAt(2)` returns `'I'`. What does `"THEOREM".charAt(1)` return?

   Answer: __________

2. *Boolean Algebra* deals with logic; it has $\wedge$ (AND), $\vee$ (OR) and $\neg$ (NOT) operators.

   $X \wedge Y$ is True only if both X and Y are True;
   $X \vee Y$ is True if X is True or Y is True (or both are True);
   $\neg X$ is True if X is False; $\neg X$ is False if X is True.

   Is this Boolean expression —

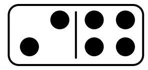
   $$(X \vee Y) \wedge (\neg X \vee \neg Y)$$

   — True or False when:

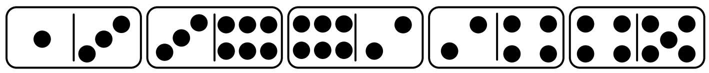
   (a) X is True and Y is True
   (b) X is True and Y is False
   (c) X is False and Y is False

   Answer: (a) _____________ (b) _____________ (c) _____________

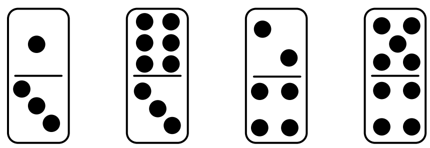
3. A *domino* is a rectangular tile with a line dividing its face into two square halves; each half has a number of *pips* on it, from 0 to 6. For example,



Dominoes form a valid chain if the touching halves of any two touching dominoes have the same number of pips. For example:



Given four dominoes —



— there are several ways to add one more domino and build a valid chain of five dominoes. One example is shown above. In that example we add the 6|2 domino, which has 8 pips total. What is the smallest possible total number of pips on the missing domino that lets us build a valid chain of five dominoes?

Answer: __________

4. Java is a widely used programming language. This Java code —

```
int n = 2;
int x = 1, sum = 0;

while (n > 0)
{
  sum += x;   // Add x to sum
  x += 2;
  n -= 1;     // Subtract 1 from n
}

System.out.println(sum);
```

— displays 4. What will it display if we replace `n = 2` with `n = 4`?

Answer: __________

5. Alice uses a cipher in which each letter of the English alphabet is encoded as a number: 1 for A, 2 for B, 3 for C, and so on. To make her cipher harder to crack, Alice writes each number in the *binary* (base-2) number system. (The binary system uses only 2 digits: 0 and 1.) To make all the letter codes the same length in binary, Alice adds zeros on the left side of each code, where needed, so that each code has five digits:

A	00001
B	00010
C	00011
D	00100
E	00101
F	00110
...	...
Z	11010

For example, the code for `JACK` will be `01010 00001 00011 01011`. What will be the code for `JILL`?

Answer: ____

6. You have two empty glasses, a three-ounce glass and a five-ounce glass. You are allowed to perform three operations:

   1. Fill a glass with water.
   2. Empty a glass into the sink.
   3. Pour as much water as possible from one glass into the other (until the first glass is empty or the second glass is full) .

   You need to get exactly one ounce in the three-ounce glass, with the five-ounce glass empty. What is the smallest possible number of operations needed?

   Answer: __________

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Answers and Solutions

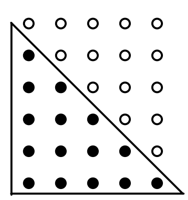
Meet 1

1. 81
2. 55
3. Line 4
4. 1024
5. C (Or "Yay, I won")
6. 38

Solutions:

1. If you follow the pattern, you see that each response is three times the previous one. Actually, the dialog comes from Python; in Python, `a**n` means *a* raised to the *n*-th power.

2. The sequence continues: $T_6 = 21, T_7 = 28, T_8 = 36, T_9 = 45, T_{10} = 55$. The numbers are called *triangular* because they represent the number of dots in a triangle. For example:

   As you can see in the above picture, $T_n = \frac{n(n+1)}{2}$.

3. Line 4 should be:

   ```
   forward(60)
   ```

   to move the turtle to the "inner" corner of the "L."

4. The sum in the third row is 8. The next row, not shown, will be 1, 4, 6, 4, 1, and the sum in that row is 16. It looks like the sums of the numbers in consecutive rows are consecutive powers of 2. (This is not really surprising, because a number in a given row is the sum of two numbers in the previous row.) So keep multiplying by 2: Row 5 — 32, 64, 128, 256, 512, Row 10 — 1024. In general, the sum in the *n*-th row is 2^n.

   The numbers in Pascal's Triangle are "choose-n-k" numbers (the number of ways to choose *k* objects out of *n* different objects). These numbers are also called *binomial coefficients*, because they are the coefficients at the powers of *x* in the expansion of $(x+1)^n$. The sum of the coefficients in the *n*-th row is the value of $(x+1)^n$ at $x = 1$.

5. Since `gameOver` is `false`, we immediately move on to the `else` clause. Since `myScore` is 5 and `yourScore` is 3, the first condition is `true`.

6. We can remove any pair of edges, except 13+10 and 17+18. We need to select among the remaining eight pairs the one that gives the greatest sum. It is 13+18=31. The sum of the remaining weights is 69 – 31 = 38.

Meet 2

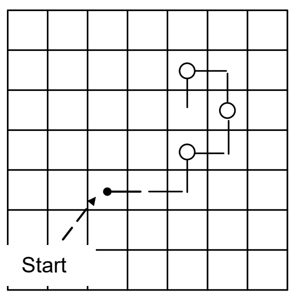
1. 24
2. 1
3. B
4. 19
5. `1D6B`
6. 48

Solutions:

1. `5*3 = 15; 15 - 3 = 12; 12*2 = 24`

2. `13 % 5 = 3`; `13 % 3 = 1.`

3.



4. The smallest *n* with $s(n) \geq 10$ is 19. $s(19) = 10$ and $s(s(19)) = 1$.

5. `1110101101011 ==> 0001 1101 0110 1011 ==> 1D6B`

6. This expression is equivalent to `4*((1 + 3)*(5 - 2))`

Meet 3

1. `'H'`
2. false, true, false
3. 3
4. 16
5. `01010 01001 01100 01100`
6. 5

Solutions:

1. In Java, the `charAt(k)` *method* (function) of strings returns the *k*-th character in the string. However, in Java and most other programming languages, characters are counted starting from 0. So `"THEOREM".charAt(1)` returns `'H'`.

2. For this expression to be True, $(X \vee Y)$ must be True, so at least one of X and Y must be True; and $(\neg X \vee \neg Y)$ must be True, so at least one of X and Y must be False. Bottom line: X and Y must have different values: one should be True and the other False. Boolean Algebra includes a secondary operator, XOR ("exclusive OR"): $X \oplus Y$ is True if X is True or Y is True, but not both. Thus the expression in the question is equivalent to $X \oplus Y$.

3. The 1|2 domino connects the 6|3·3|1 and 2|4·4|5 segments. It has the smallest possible number of pips to complete the chain; 0|0, 0|1, 0|2, or 1|1 do not work.

4. The code computes the sum of the first several odd numbers. 1 + 3 = 4; 1 + 3 + 5 = 9; 1 + 3 + 5 + 7 = 16. Notice that the sum of the first n odd numbers is equal to n^2.

5. The code for J is given in Jack, the code for I is one less (compare the codes for B and A). (Alternatively, you can compute the codes for F, G, H, and then I .) The code for L is next after the code for K, given in JACK (compare the codes for C and D).

6.