Continental Mathematics League

2015 - 2016

Computer Science Contest

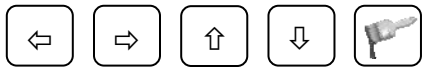
Grades 3-5

The contest consists of three "meets." Each meet has six questions for 30 minutes.

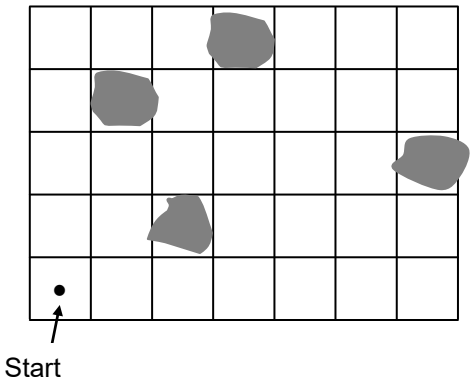
Note: Some questions have multiple answers. All parts of the question must be answered correctly for a student to receive credit for the problem.

Computer Science Meet 1 — January 14, 2016

1. Pink Painter is a programmable robot. She can move left, right, up, and down, and paint the square she is on. These five commands are represented by the blocks



Pink Painter started in the lower left corner, drew the picture below, and returned to the starting point.



What is the smallest possible number of commands needed to accomplish that?

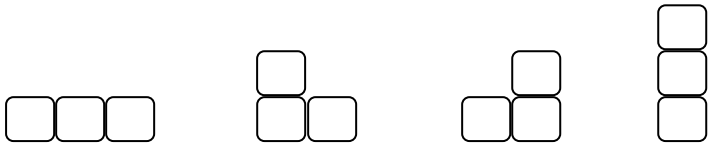
Answer: _____

2. Fill in the blank:

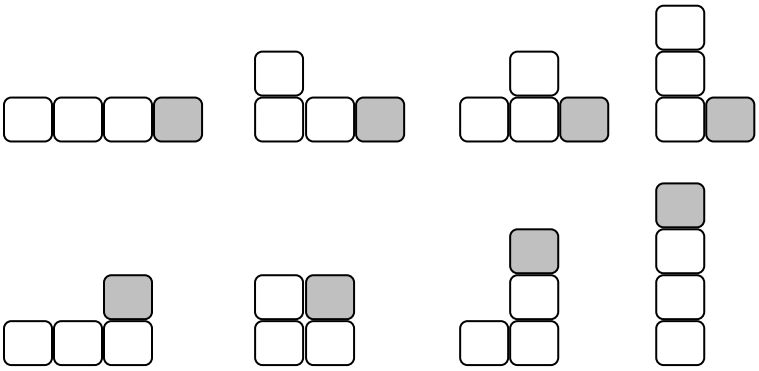
0001	1
0010	2
0011	3
0100	4
1000	8
1001	9
10001	____

Answer: _____

3. A "castle" is made of square blocks in such a way that there are no gaps in the "ground" (bottom) row, and each block either stands on the ground or on another block. You can make four different "castles" with three blocks:



With four blocks, you can make eight different "castles":



How many different castles can you make with six blocks?

Answer: _____

4. Python is a popular programming language. Here is a very short program in Python:

```
n = int(input("Enter a number: "))
print(sum(range(1, n+1)))
```

It asks the user to enter a number. If the user enters 1, the program displays 1; if the user enters 2, the program displays 3; if the user enters 3, the program displays 6. What does the program display if the user enters 5?

Answer: _____

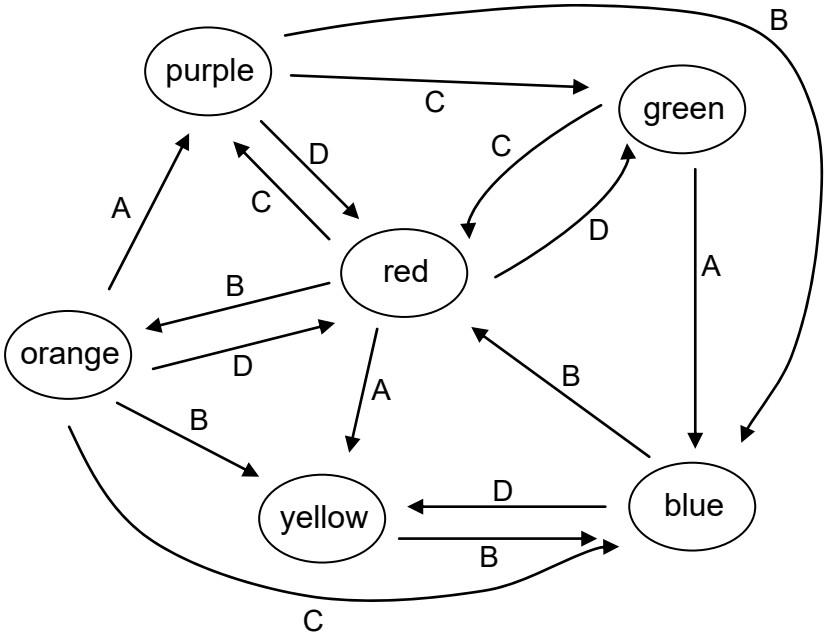
5. In the programming language Java, `&&` means "and" and `||` means "or". The following snippet of Java code uses these two *operators*.

```
summer = true;
winter = false;
night = true;
day = false;

t = 0;  // Set the value of t to 0

if (winter && (night || day))
  t = 1;
else if (summer && day)
  t = 2;
System.out.println(t);  // Print the value of t
```

What is printed when this code runs?

Answer: _____

6. Chameleon ("Cam" for short) is a *state machine*. State machines are often used to model behavior of devices and computer programs. In this case, Cam's "state" is its color. Cam receives a string of letters that serves as Cam's list of commands to follow. Cam changes its color, depending on its current state and the next letter in the string. The diagram below shows Cam's "brain": a letter by an arrow tells how Cam will change its color if that letter is received next.



For example, if Cam starts in the orange state and receives "AD", it will first change its color to purple, then to red. In which color will Cam end up if it starts in orange state and receives "ADBADDAD"?

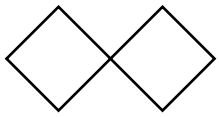
Answer: _____

Computer Science Meet 2 — February 11, 2016

1. Python is a popular programming language. It has commands for "turtle graphics." This program (with a smudge over two commands) —

```
art = Turtle("turtle")  # Start in the middle,
                        #   facing to the right
art.right(45)    # Turn right 45 degrees
art.pendown()    # Prepare to draw
art.forward(40)  # Move forward 40 units
art.left(90)
art.forward(40)
art.left(90)
art.forward(40)
art.left(90)
a[redacted]
art.forward(40)
art.right(90)
art.forward(40)
art.right(90)
art.forward(40)
art.hideturtle()
```

— makes this picture:



Which two commands are hidden under the smudge?

Answer:

_____

_____

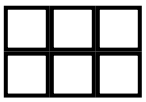
2. Computers store numbers using the *binary* number system, as strings of zeros and ones. For example, 00101110 represents 46. But binary numbers with many digits are hard for people to read. So computer programmers sometimes use the *hexadecimal* numbers, which are easier to read and easy to convert to binary. The hexadecimal number system uses 16 "digits": 0 - 9, `A`, `B`, `C`, `D`, `E`, `F`. `A` represents 10, `B` represents 11, and so on, `F` represents 15. Here are some examples of numbers written in our usual *decimal* system and in hexadecimal (*hex* for short):

Decimal	Hex
5	5
8	8
9	9
10	`A`
11	`B`
...	...
15	`F`
16	10
17	11
...	
25	19

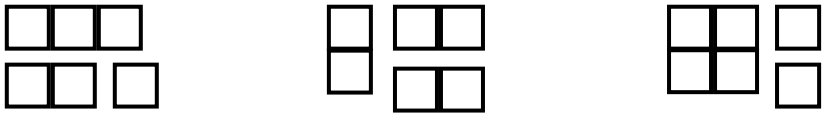
How is 30 written in hex?

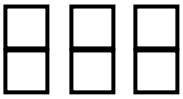
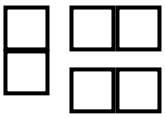
Answer: _____

3. A chocolate bar is made of squares separated by grooves. For example, here is a 3 by 2 bar:



You first break the bar along a groove into two pieces, then you break one of the pieces along a groove into two. So you end up with three pieces. If only the sizes of the pieces matter, there are three different ways of breaking a 3 by 2 bar into three pieces:



( and  are considered the same.)

In how many different ways can you break a 4 by 2 bar into three pieces?

Answer: _____

4. Java is a widely used programming language. Here is a snippet of Java code:

```
int k = < a number entered by the user >
int p = 1;

while (k > 0)
{
  System.out.print(k + " " + p + " ");
  p *= 2;
  k--;
}
```

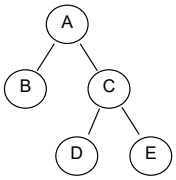
If the user enters 3, the program displays

```
3 1 2 2 1 4
```

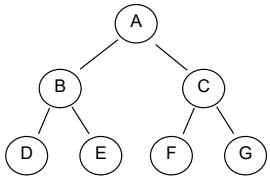
What does the program display if the user enters 4?

Answer: _____

5. In computer science, a *tree* structure is sometimes used to hold data in its *nodes*. A tree is usually drawn upside down, with the *root* node at the top. For example:

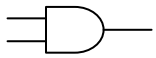


Here the root node holds A. Scrat collects the letters from the tree, moving from one node to another. His next move is determined by the same *algorithm* (method), regardless of the node Scrat is in: pick all the letters from the left branch, then pick the letter in the current node, then pick all the letters from the right branch. If Scrat starts at the root node of the above tree, he will collect the letters in this order: B, A, D, C, E. In what order will Scrat collect the letters if he starts at the root node of this tree:



Answer: _____

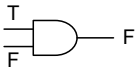
6. An electronic circuit can be made of *gates*. The AND gate is drawn like this:



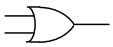
It takes two inputs, which can be *T* (true) or *F* (false), and produces one output, *T* or *F*. If both inputs are *T*, the output is *T*, otherwise the output is *F*, as shown in the following table:

Inputs:		Output from AND:
T	T	T
T	F	F
F	T	F
F	F	F

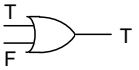
For example:



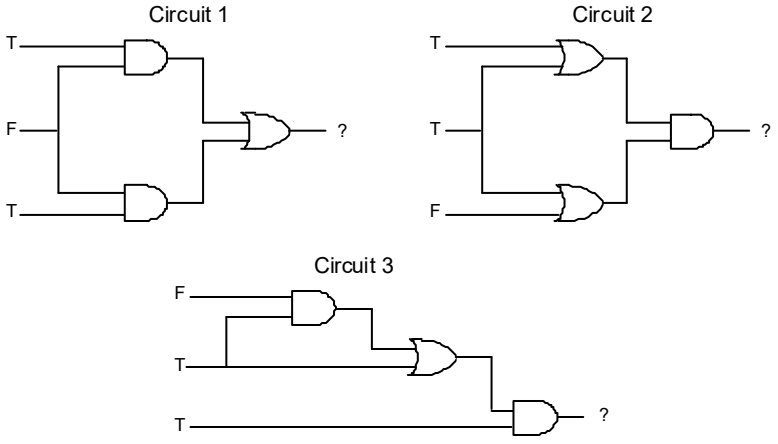
The OR gate is drawn like this:



If at least one of the inputs of the OR gate is *T*, then its output is *T* . If both inputs are *F*, then the output is *F*. For example:



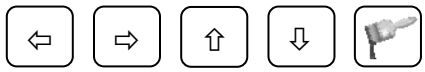
What are the outputs of these three circuits with given inputs:



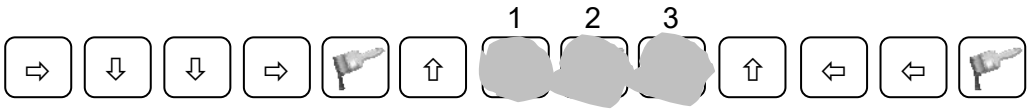
Answer: Circuit 1 _____ Circuit 2 _____ Circuit 3 _____

Computer Science Meet 3 — March 3, 2016

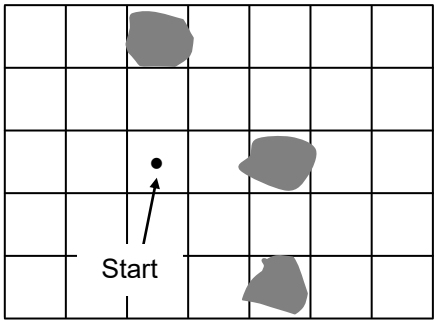
1. Pink Painter is a programmable robot. She can move left, right, up, and down, and paint the square she is on. These five commands are represented by the blocks



Pink Painter was given this program (in which three blocks are hidden):



She painted this:



What commands are not shown?

Answer: 1:_____ 2: _____ 3: _____

2. We use 10 digits, 0, ..., 9, to represent numbers, because we have 10 fingers. This is called *decimal* or *base 10* number system. If we had only three fingers, we would represent numbers in *base 3* system, using only three digits, 0, 1, and 2. For example:

Base 10	Base 3
1	1
2	2
3	10
4	11
5	12
6	20
...	...
9	100

What is the base-3 representation of 14?

Answer: _____

3. Python is a popular programming language. Here are three user inputs and computer responses generated by the Python *interpreter*:

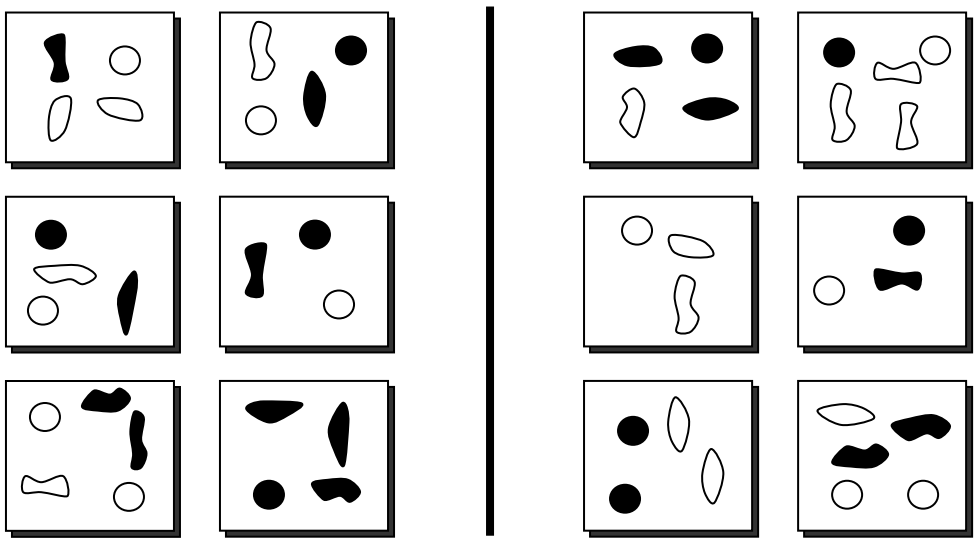
```
>>> print(2 * 2)
4
>>> print(2 * '2')
'22'
>>> print(2 * [2])
[2, 2]
```

What will the computer respond to this input:

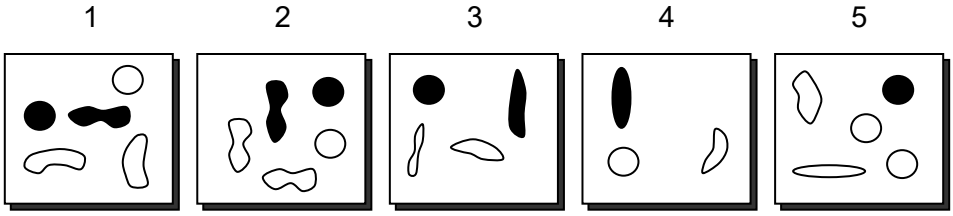
```
>>> print(2 * [2 * '2'])
```

Answer: _____

4. Mikhail Bongard, a Russian computer scientist, invented a kind of puzzle that a computer was supposed to solve. Here is one:



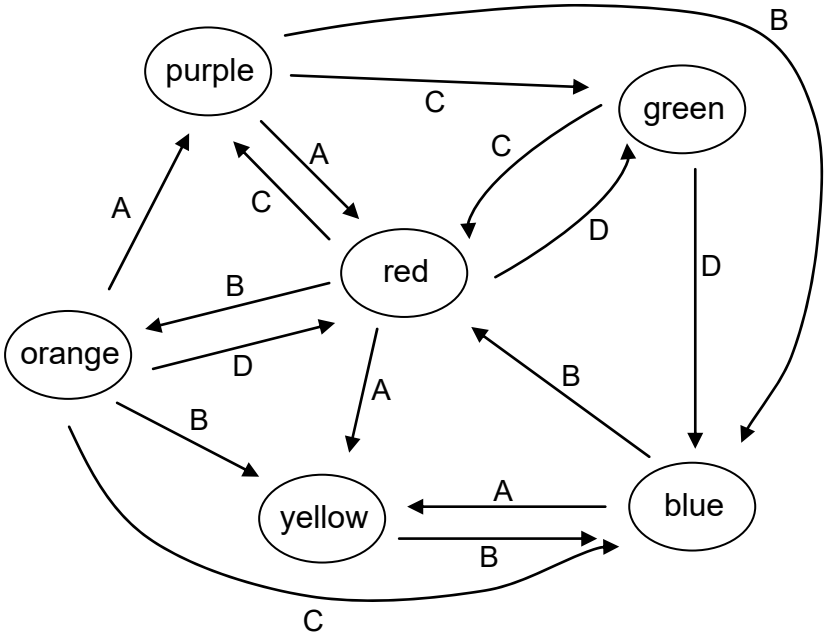
The six pictures to the left of the dividing line are in some way different from the six pictures to the right. Here are five more pictures:



Three of them belong with the left-side pictures. Which ones?

Answer: _____, _____, and _____

5. Chameleon ("Cam" for short) is a *state machine*. State machines are often used to model behavior of devices and computer programs. In this case, Cam's "state" is its color. Cam receives a string of letters that serve as Cam's commands. Cam changes its color, depending on the next letter in the string and on its current state. The diagram below shows Cam's "brain": a letter by an arrow tells how Cam will change its color if that letter is received next.



However, Cam can't process all commands: sometimes it gets stuck. Which three of the following five commands are "legal," that is, Cam can handle them if it starts in orange?

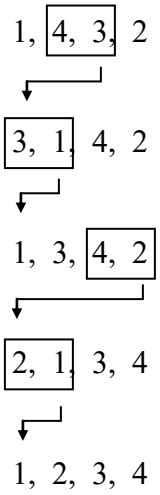
1. AAABBB 2. BBBCD 3. CABBC 4. DDAAB 5. DCADCA

Answer: _____, _____, and _____

6. Suppose we want to arrange a list of numbers in order. (This operation is called *sorting*.) We want to use the following Silly Sort *algorithm* (precise method):

Step 1: Starting from the left, find the first pair of numbers that are out of order
Step 2: Move the smaller number of the pair to the beginning of the list
Step 3: Repeat Steps 1 and 2 until the list is *sorted*

For example, if we apply this algorithm to the list 1, 4, 3, 2, four "moves" will be needed to sort the list:



How many moves are needed to sort the list 1, 2, 5, 3, 4 using Silly Sort?

Answer: _____

Computer Science Answers and Solutions

Meet 1- January 14, 2016

1. 24
2. 17
3. 32
4. 15
5 0
6 yellow

Solutions:

1. 20 moves and 4 paint commands are needed. The two painted squares closest to the origin should be visited first and last. (The smallest number of moves is actually the same as needed to trace the border of the rectangle.)

2. 10000 binary is 16. 10001 binary is 17.

3. As can be discerned from the given example, adding one block doubles the number of possible castles (because one extra block can be added in the bottom row to the right of the castle or on top of the rightmost block in the castle).

4. The program displays the value of $1 + 2 + ... + n$. If n is 4, the program displays $6 + 4 = 10$. If n is 5, the program displays $10+5 = 15$.

5. Since `winter` is `false`, the condition in `if` is `false`; since `day` is `false`, the condition in `else` is `false`. So neither the statement under `if` nor the statement under `else` will be executed, and the value of `t` will remain 0.

6. The colors will change in this sequence: orange (start) => purple => red => orange => purple => red => green => blue => yellow.

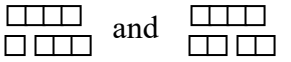
Meet 2- February 11, 2016

1. `art.forward(80)`
   `art.right(90)`
2. `1E`
3. 5
4. `4 1 3 2 2 4 1 8`
5. D, B, E, A, F, C, G
6. *F*, *T*, *T*

Solutions:

1. `art` first draws three sides of the right diamond, then cuts top-right to bottom-left with a segment of double length, then paints the remaining three sides of the left diamond.

2. 25 is hex `19`, 26 is hex `1A`, ..., 29 is hex `1D`, 30 is hex `1E`.

3. There are two ways to break the bar if the first break is along the long groove:
    and .
   There are three ways to break the bar if the first break is along a short groove:
    ,  and  .

4. It is easier to interpret the output if you look at every other number:

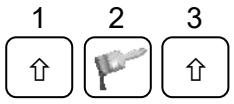
   `4 1 3 2 2 4 1 8`

   In each *iteration* (pass) in the `while` *loop*, the values of `k` and `p` are printed, then `k` is decreased by 1 and `p` is doubled. When `k` is 0, the iterations stop.

5. Scrat first collects the letters from the left branch: left, root, right, that is D, B, E. Then the root, A. Then the right subtree, F, C, G. (This algorithm is called *inorder traversal* of a *binary tree*.)

6. In Circuit 1, the output of both AND gates is *F*, so the output of the OR gate is *F*. In Circuit 2, the output of both OR gates is *T*, so the output of the AND gate is *T*. In Circuit 3, the output of the leftmost AND gate is *F*, but the output of the middle OR gate is *T* and so the output of the rightmost AND gate is *T*.

Meet 3- March 3, 2016

1. 

2. 112
3. `['22', '22']`
4. 2, 3, and 4
5. 1, 3, and 5 (or `AAABBB`, `CABBC`, and `DCADCA`).
6. 12

Solutions:

1. The bottom blob is painted first. It takes two ⇧ commands to get to the blob above it, so the first hidden command is ⇧ and the second is "paint." It takes two ⇧ and two ⇦ commands to get from there to the third (topmost) blob. ⇧ and ⇦⇦ commands are shown, so the third hidden command must be ⇧.

2. 14 = 9 + 3 + 2. 9 in base 3 is 100; 3 in base 3 is 10; 2 in base 3 is 2.

3. In Python, *n* times a string in quotes produces a new string in which the contents of the original string is repeated *n* times. For example, `3 * '2'` produces `'222'`. *n* times a list in brackets produces a new list in which the original list is repeated *n* times. For example, `3 * [2]` gives `[2, 2, 2]`. The expression in the question includes both these operations: first the contents of `'2'` is repeated twice, giving `'22'`, then `'22'` in the list `['22']` is repeated twice, giving `['22', '22']`.

4. The pictures on the left have at least one "vertical" solid black blob.

5. `BBBCD` is invalid because `BBBC` ends in purple, from which there is no `D` arrow. `DDAAB` is invalid because `DD` leads to green, from where there is no `A` arrow.

6. 1,2,5,3,4 => 3,1,2,5,4 => 1,3,2,5,4 => 2,1,3,5,4 => 1,2,3,5,4 => 4,1,2,3,5 => 1,4,2,3,5 => 2,1,4,3,5 => 1,2,4,3,5 => 3,1,2,4,5 => 1,3,2,4,5 => 2,1,3,4,5 => 1,2,3,4,5